# TTK2 – Pålitelighetsanalyse av sikkerhetsfunksjoner

## **Seminar 3** – Fault tree analysis

Versjon 05.08.2026

Beklager: Litt norsk og litt engelsk….

2026

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https://www.itk.ntnu.no/emner/fordypning/TTK2

# Om bruk av innhold fra slidene



La de gjerne inspirere, men husk å referere meg om du bruker noe fra presentasjonen direkte

# About fault tree analysis (FTA)

**Widely used!** Nuclear, aviation, aerospace railway, process industry, …

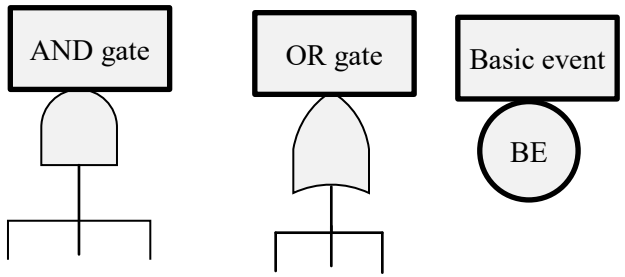
Why?

- Can incorporate **all kind of events**
- **Graphical** – creation of a fault tree with TOP event, gates, and basic events
- **Intuitive** – graphics easy to understand for non-experts
- **Qualitative** – determining the minimal cutsets
- **Quantitative** – calculating probability or frequency of TOP event
- Can incorporate **dependencies**


**Events**
(component failures, environmental events, human errors, command faults) **and** their **logical combinations** leading to TOP event

# FTA is a deductive (top down) analysis method

**Example of gates**

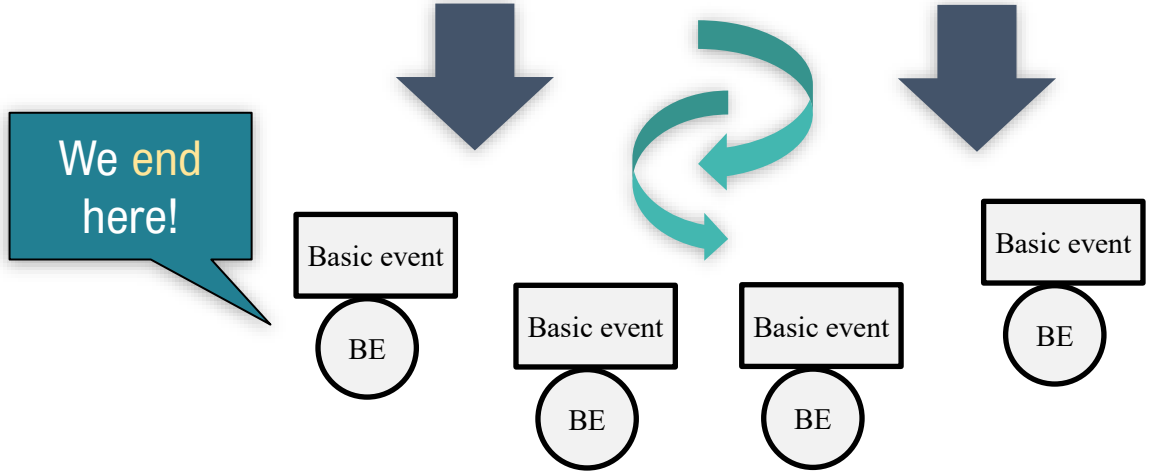



**TOP event:**
**CRITICAL/HAZARDOUS** EVENT OR SYSTEM FAILURE

We start here!

We then, iteratively ask (and model): What events, alone or in combination can lead to the event at the level above?

We end here!

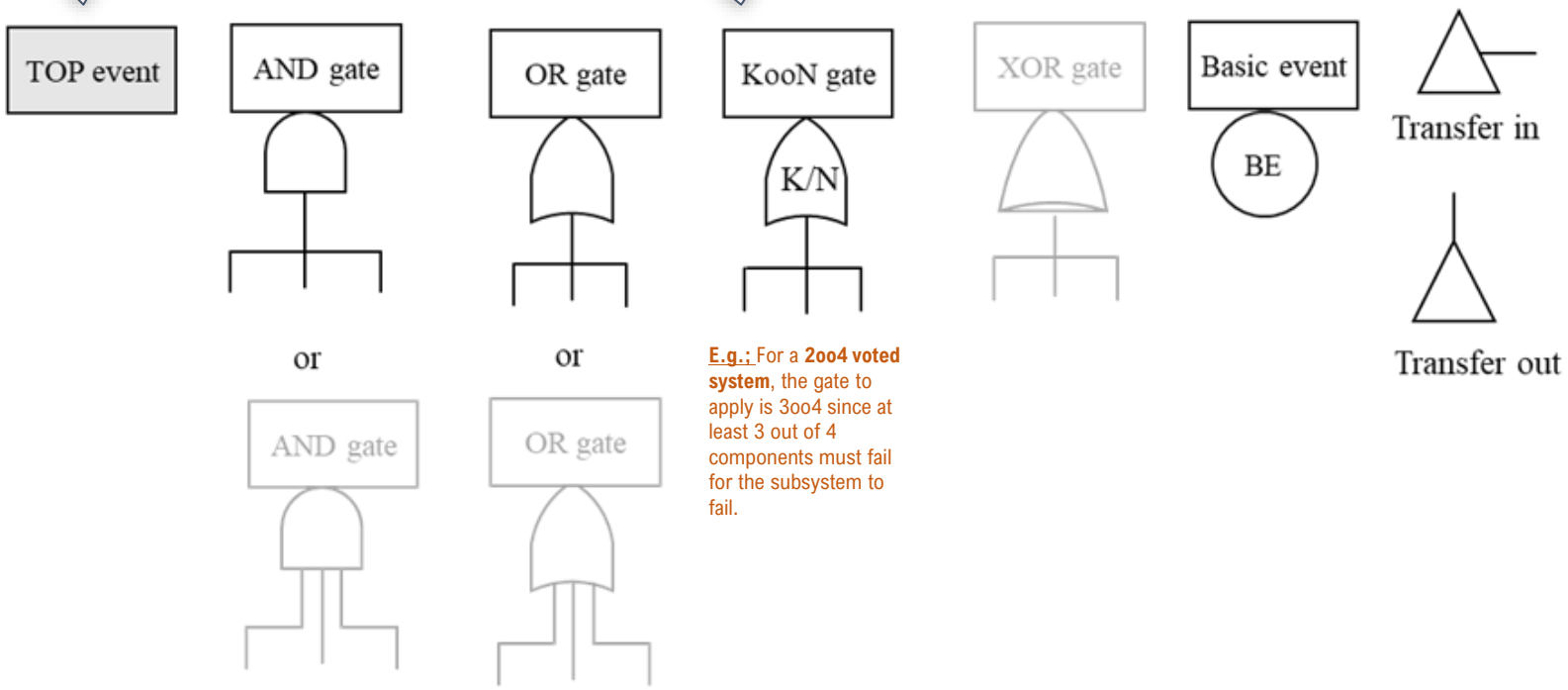



Lowest level (of each branch) contains only basic events!

# FTA symbols

Always placed on the TOP of the fault tree: The failure or hazardous situation that we want to model

Focus is **failure**, not functioning! For a KooN voted system the corresponding gate becomes **(N-K+1)ooN** for failure




**E.g.:** For a **2oo4 voted system**, the gate to apply is 3oo4 since at least 3 out of 4 components must fail for the subsystem to fail.

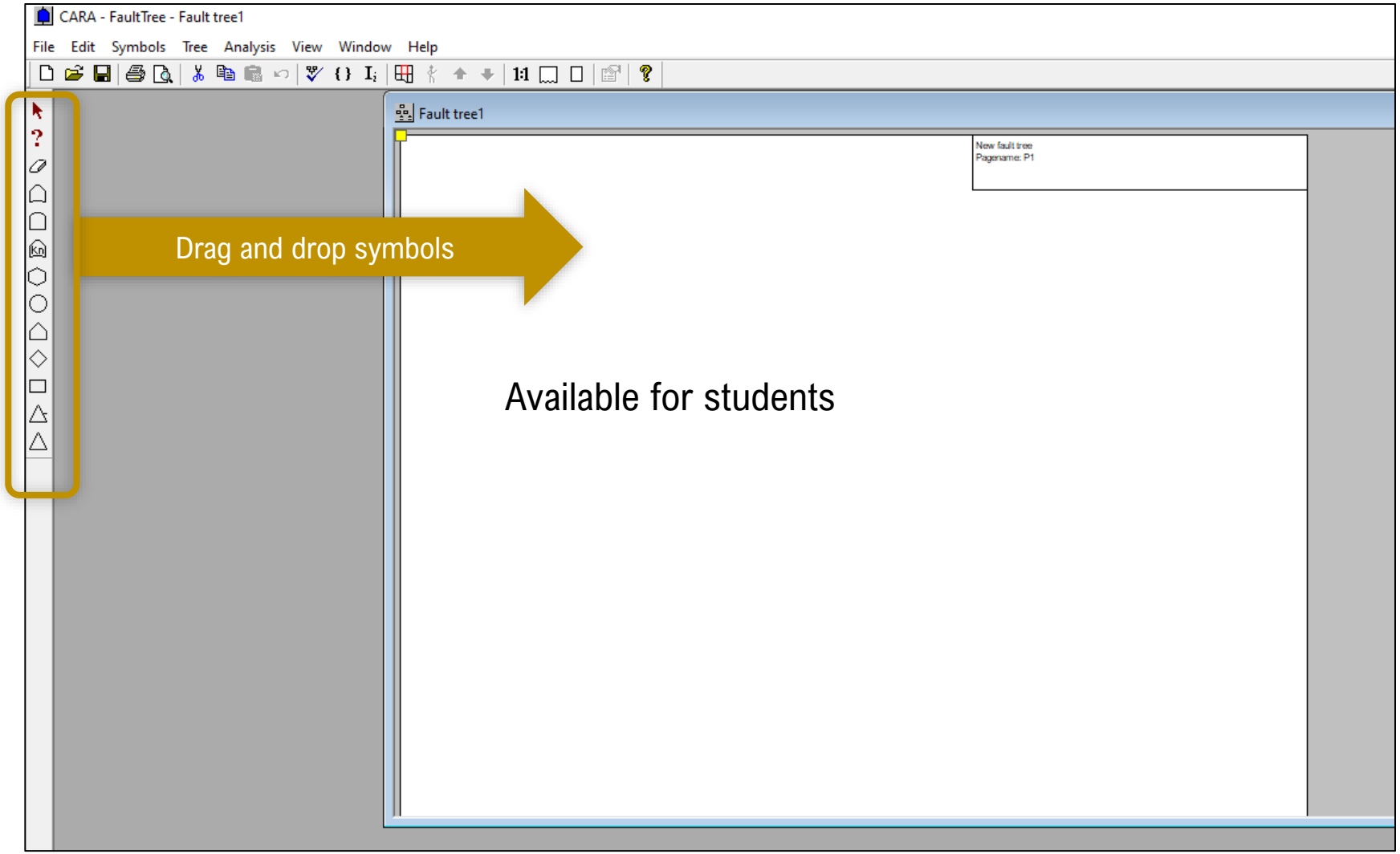
Symbols are applied to build fault trees – with or without the use of a software

# Software: Option CARA-FT

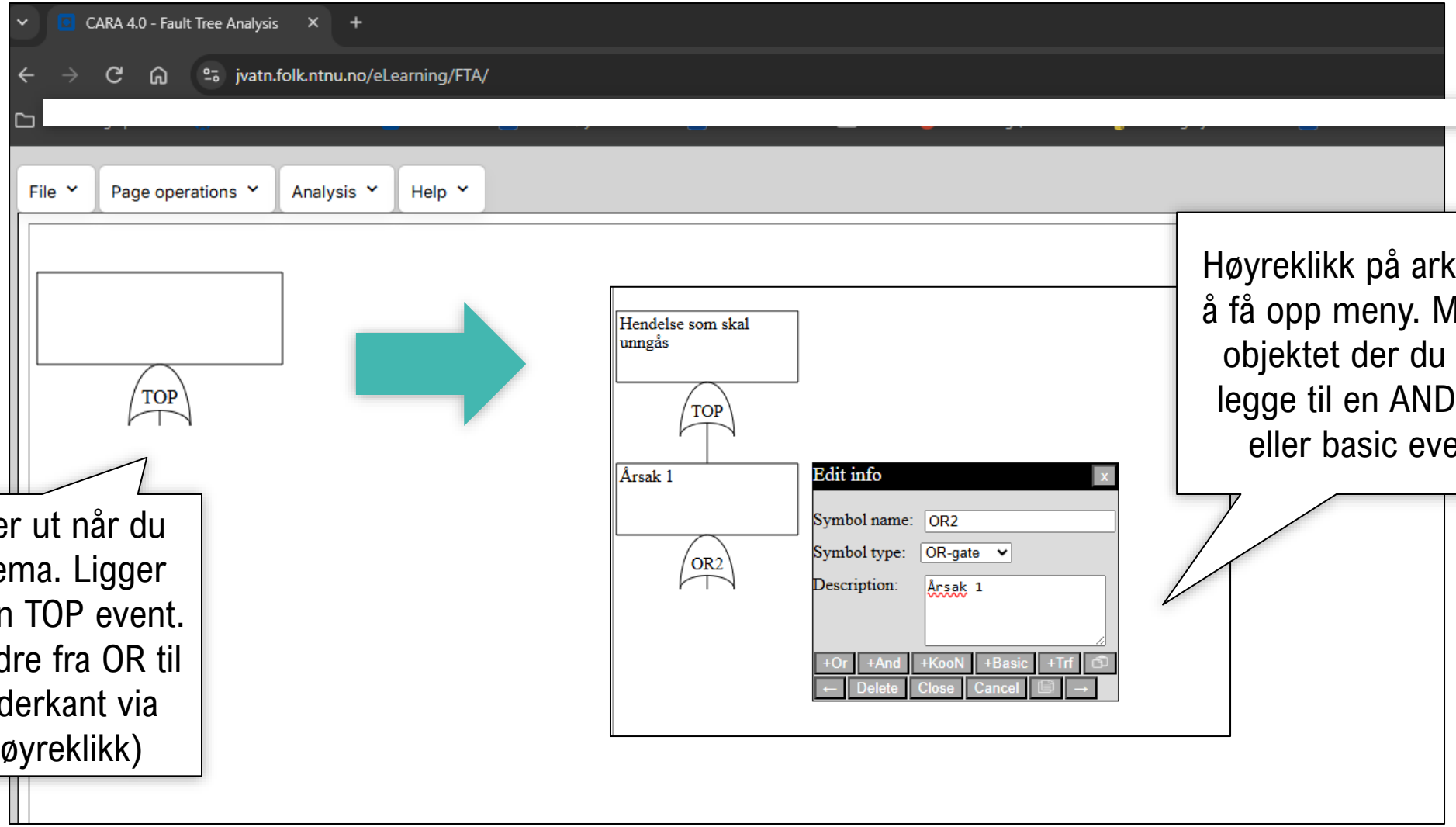
(Short demo in lecture)

| CARA-FT | Tilgjengelig via/fra | Kommentar |
|---|---|---|
| PC version | https://jvatn.folk.ntnu.no/eLearning/CaraFaultTree/CARAFaultTree.htm | Fungerer ikke med MAC<br>Må "enable" makroer for at det skal fungere |
| Web version | https://jvatn.folk.ntnu.no/eLearning/FTA/ | Programmet kan "henge seg litt opp". Her er gode tips for å komme rundt det:<br>• Run as «**Inprivate window**» (CTRL+SHIFT+N)<br>• As needed: Delete cashed images and files with CTRL+SHIFT+DEL.<br>  • For at least 24 hours, or more considering when tool as used last time.<br>  • Make sure that all AND and OR gates have unique Ids (must be updated manually when sections of the tree is copied). Refresh page afterwards!<br>• For decimals in data entries (basic events): Use «.», not «,» |

# CARA FT (PC versjon)



# CARA-FT (web-versjon)

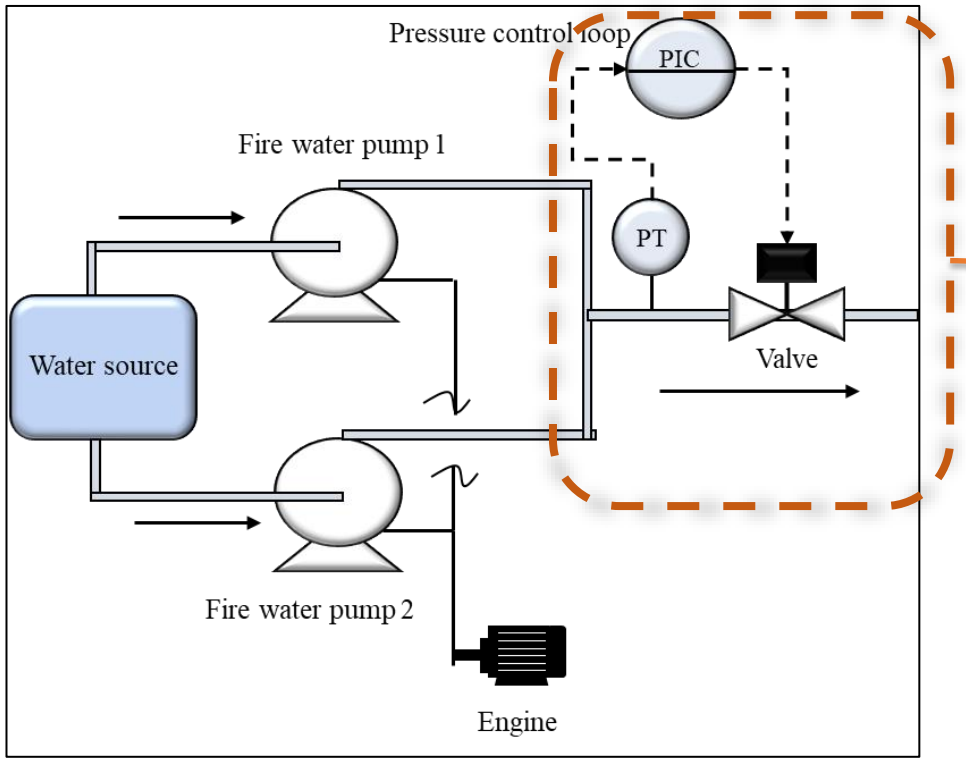


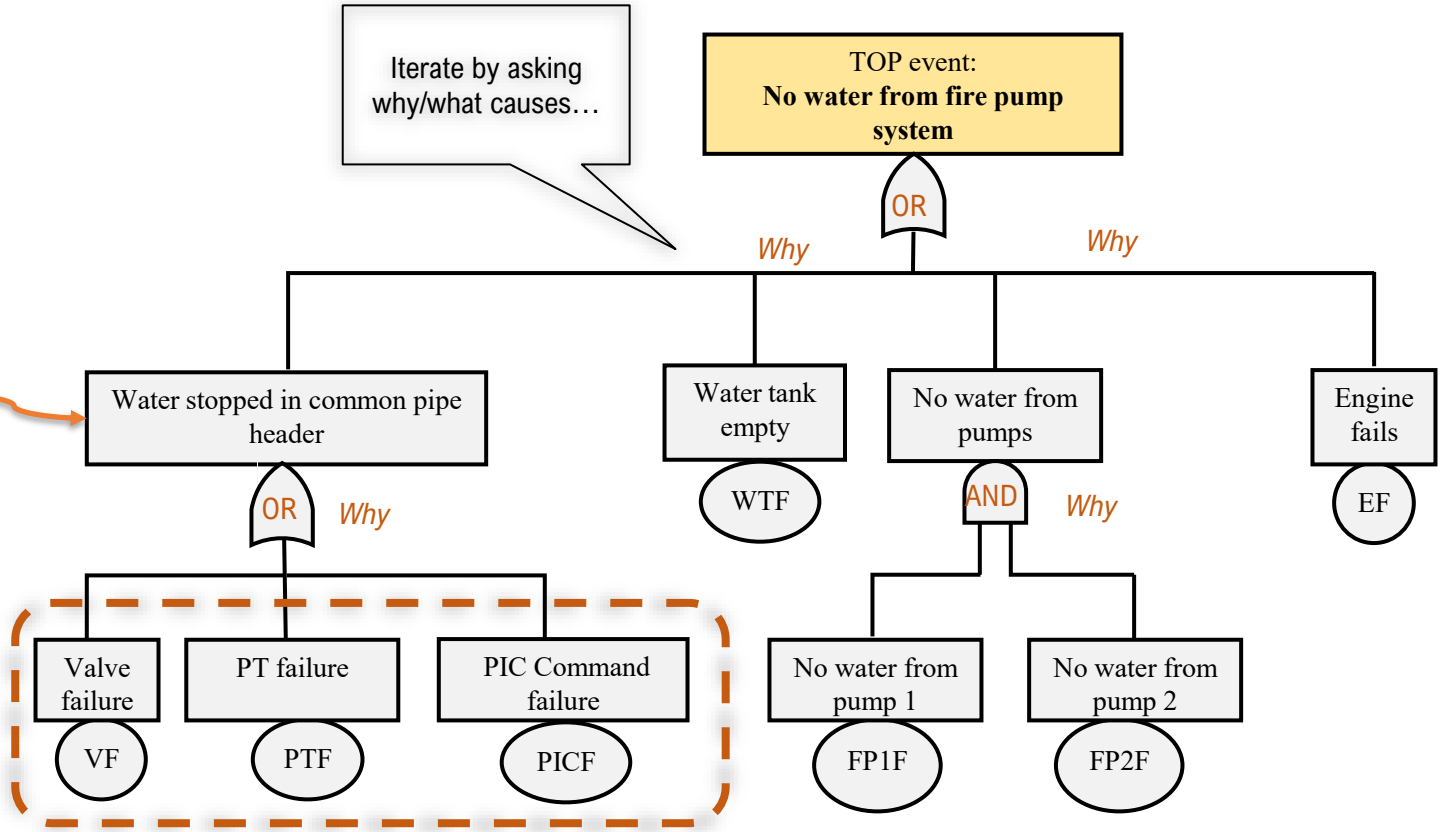

Slik det ser ut når du åpner skjema. Ligger alltid klar en TOP event. Mulig å endre fra OR til AND I underkant via meny (høyreklikk)

Høyreklikk på arket for å få opp meny. Marker objektet der du skal legge til en AND, OR eller basic event

# Building a fault tree

## Study case: Fire water system



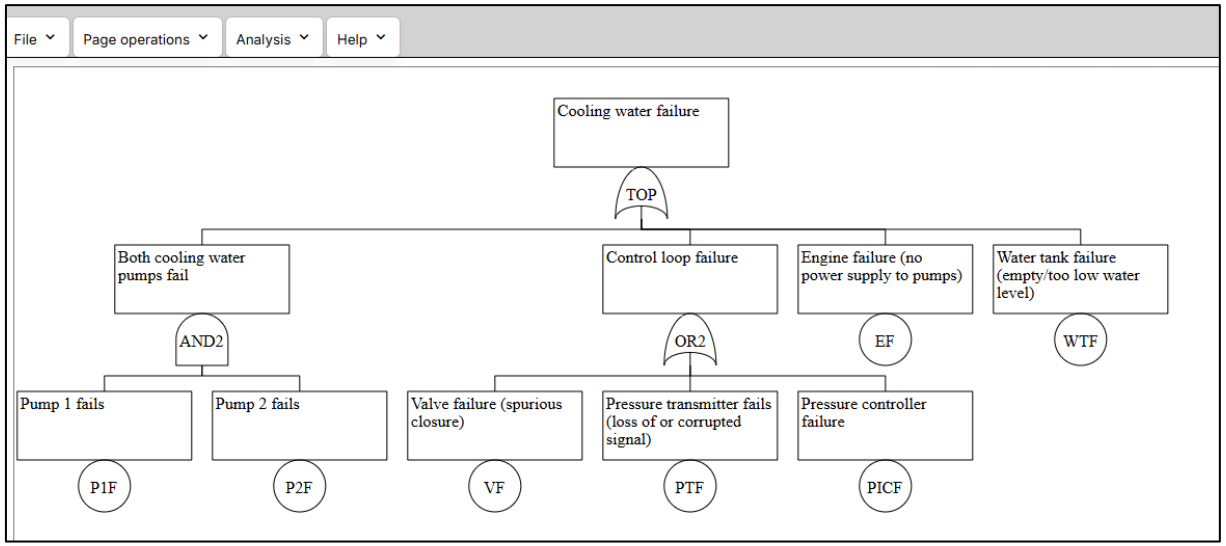



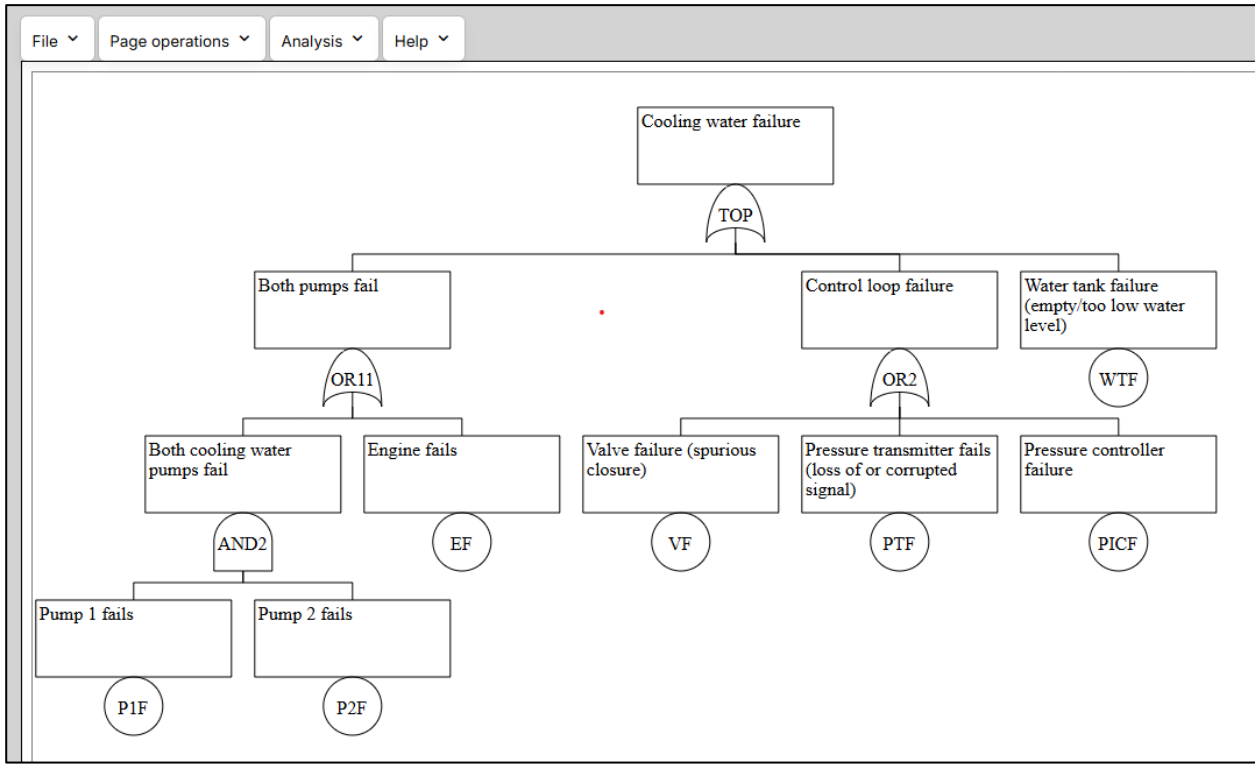

VF: Valve failure, PTF: PT failure, CF: Command failure, FPxF: Firepump nr x failure, EF: Engine failure

# Mange veier til Rom … og til hvordan et riktig feiltre vises

- Det er mange måter å utforme et riktig feiltre på
- Så lenge sammenheng primærhendelse (basic event) og TOP event er riktig, så gjør det ikke noe
- Samme basic event kan gjentas så mange ganger man vil
- Når man tar ut **minimale kuttsett** (minimal cutsets) får man ryddet opp

# Eksempel


File
Page operations
Analysis
Help
Cooling water failure
TOP
Both cooling water pumps fail
Control loop failure
Engine failure (no power supply to pumps)
Water tank failure (empty/too low water level)
AND2
OR2
EF
WTF
Pump 1 fails
Pump 2 fails
Valve failure (spurious closure)
Pressure transmitter fails (loss of or corrupted signal)
Pressure controller failure
P1F
P2F
VF
PTF
PICF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AND2
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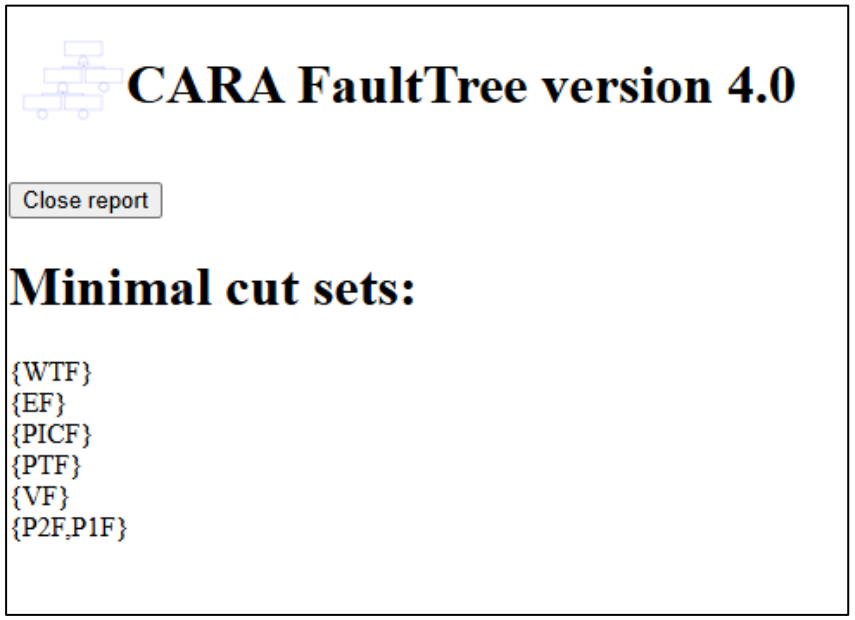
VF
PTF
PICF
Pump 1 fails
Pump 2 fails
P1F
P2F

# Qualitative analysis: Minimal cut sets

**Cut set:** A set (list of) of basic events whose (simultaneous) occurrence ensures that the TOP event occurs.

**Minimal cut set (MCS):** A cut set that cannot be reduced without loosing its status as a a cutset.




**Order** is the number of basic events inside the cutset

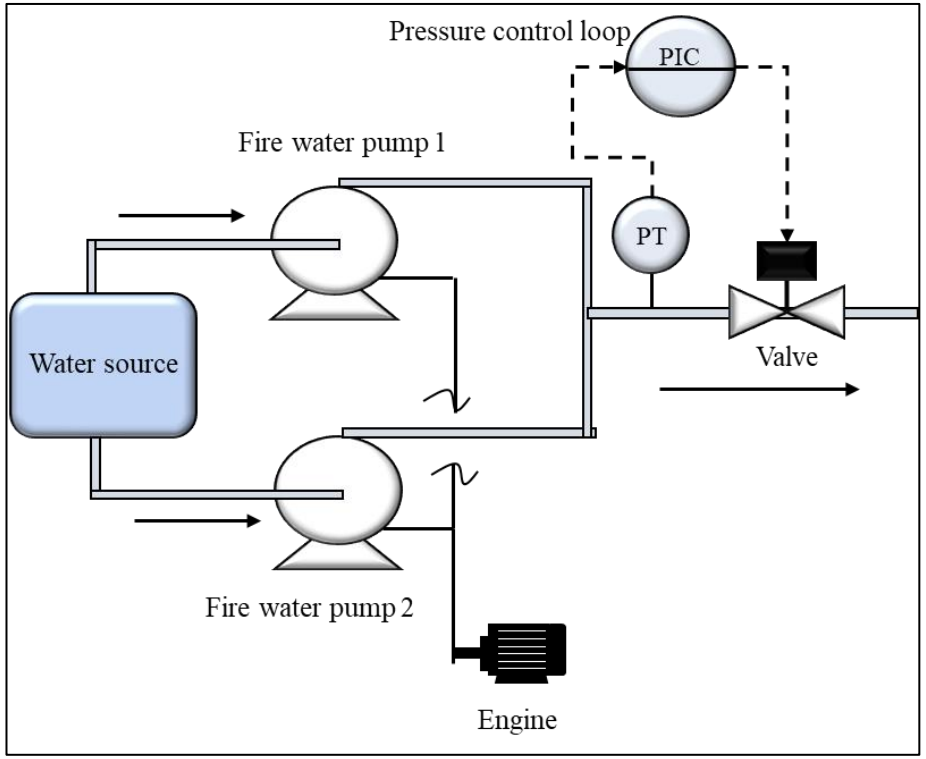
**Cut sets without CCFs for pumps**

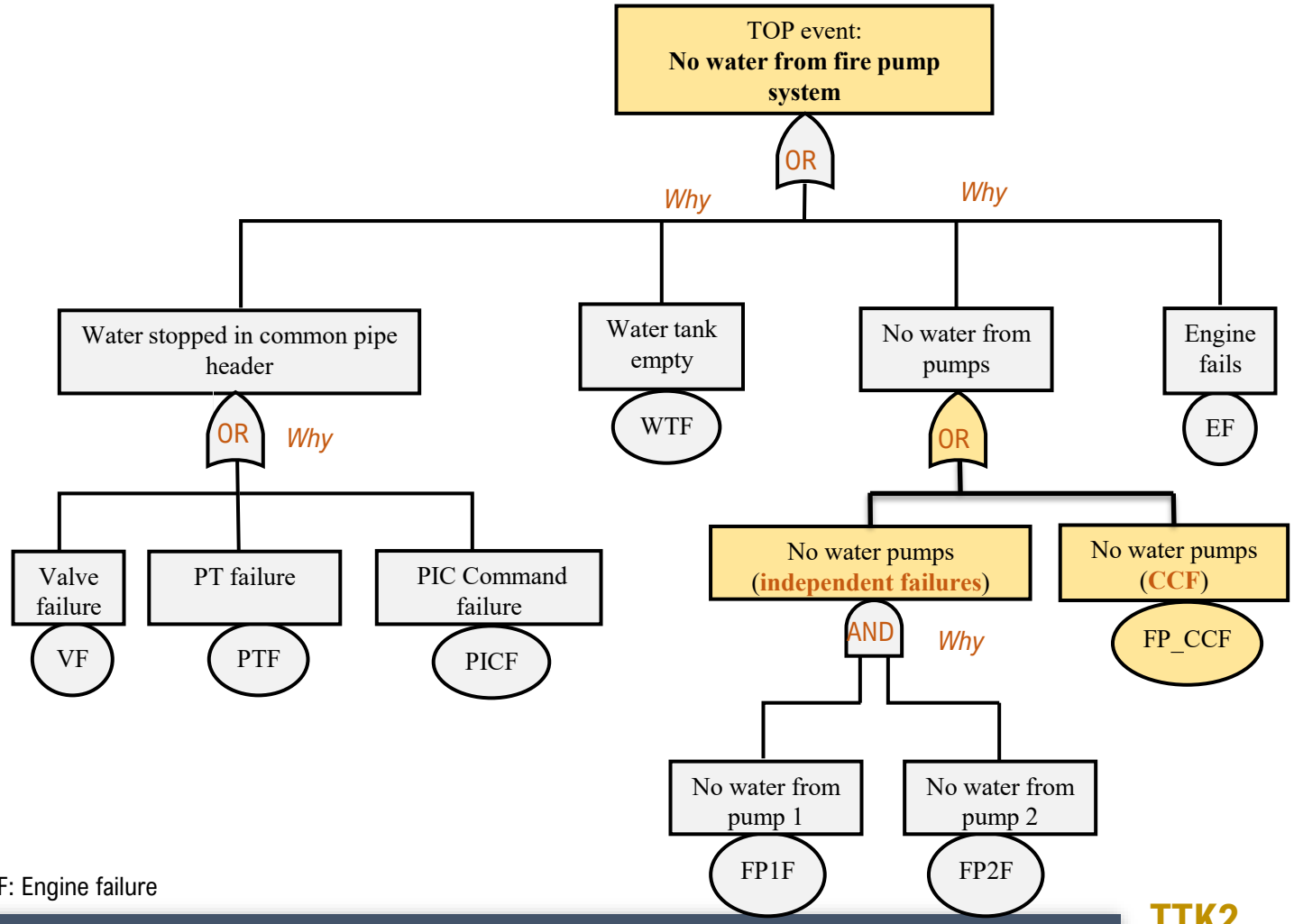
| Order of cutset | Cut sets /**Minimal cut sets*** |
|---|---|
| 1 | **[VF]*, [EF]*, [PTF]*, [PICF]*, [WTF]*** |
| 2 | [EF,VF], **[P1F,P2F]*,** [VF,PTF], [VF,PICFF],[PTF,PICFF], [EF,PTF], [EF,PICF], etc. |
| 3 | [VF,PTF,PICF], [VF,P1F,P2F], [VF,PTF,EF], {VF,EF,WTF],etc. |
| 4 | [VF,PTF,PICF,EF], [VF,PTF,P1F, P2F], [VF,PICF,EF,WTF], etc. |
| 5 | [VF,PTF,PICF,P1F,P2F], [P1F,P2F,VF,PICF,WTF], etc. |
| 6 | [VF, PTF, PICF, P1F, P2F, EF], etc. |
| 7 | [P1F,P2F,VF,PTF,PICF,EF,WTF] |

# Adding common cause failures (CCF)

**Minimal cutsets: {VF}, {EF},{PTF},{CF},{FP1F,FP2F}, {CCF}**

## Study case: Fire water system

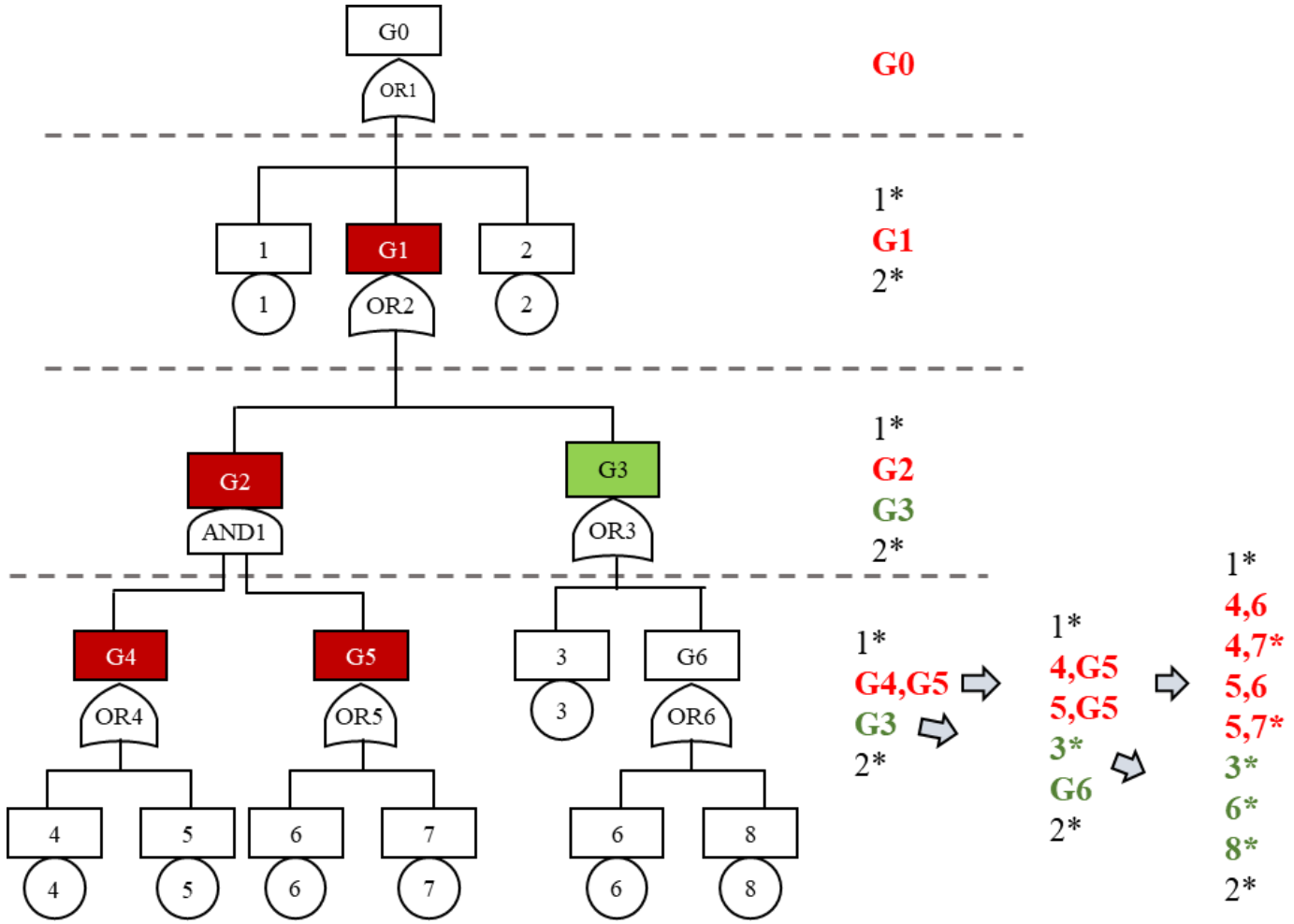



VF: Valve failure, PTF: PT failure, CF: Command failure, FPxF: Firepump nr x failure, EF: Engine failure

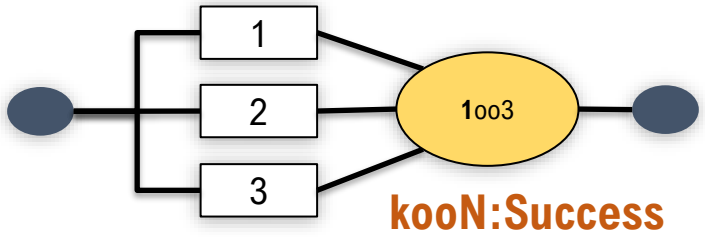
# How to identify the minimal cutsets?

- For simple fault trees, it can be done **“by hand”.**
- «All» FTA software tools include algorithms to extract minimal cut sets
- One example of algorithm implemented is the **MOCUS algorithm** from 1972.
  - OR gate - expands vertically
  - AND gate – expands horizontally




*Minimal cutsets marked with **

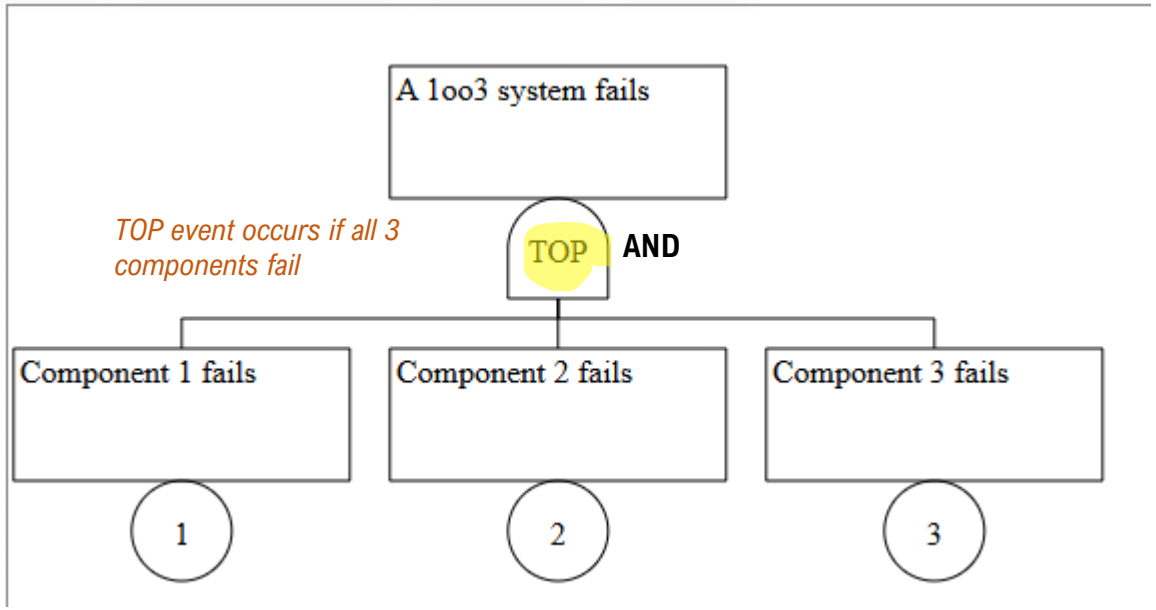
# Reliability block diagram(RBDs) vs fault trees (FTs)



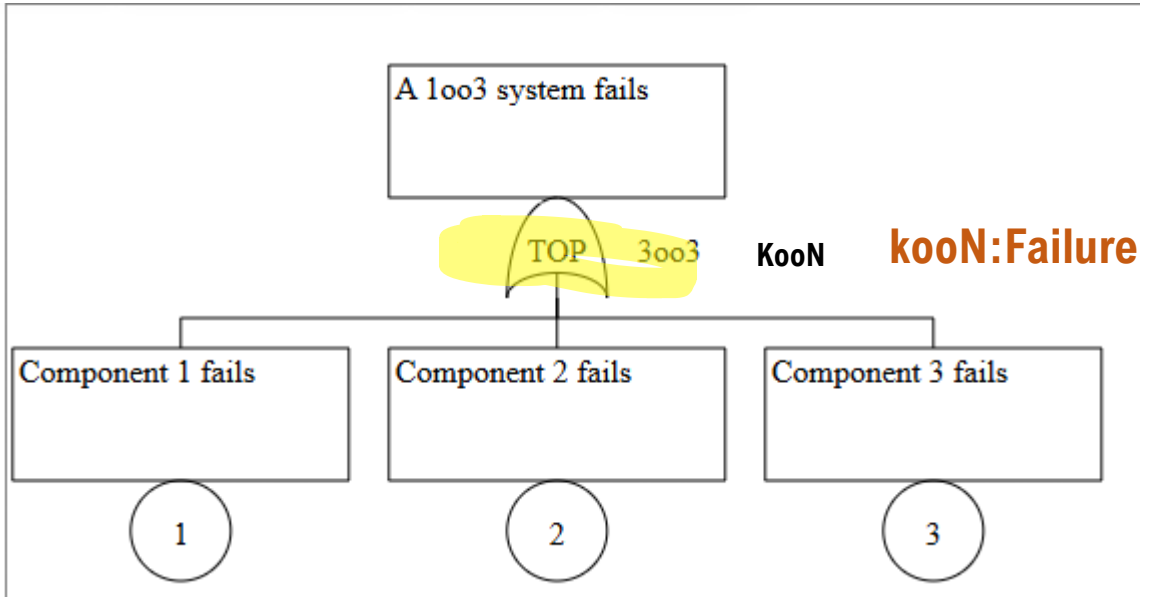

**kooN:Success**

An RBD can be converted to a FT and visa verse. While RBD is **success-oriented**, FT is ***failure-oriented***.

Variant 1

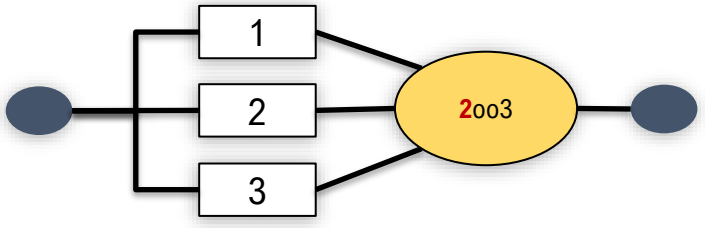



Variant 2

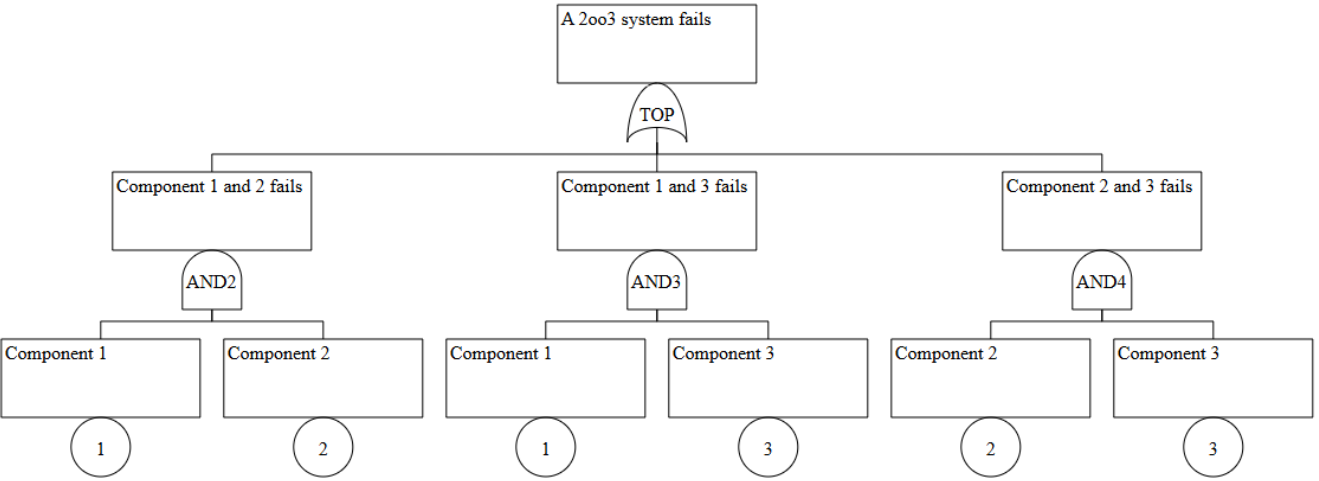



**KooN has different meanings - RBD: KooN:Success while FT: KooN:Falure**

# RBD (reliability block diagram) vs FT

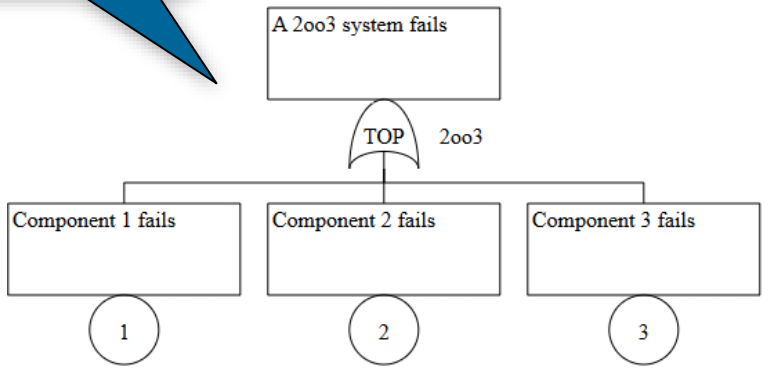



## Variant 1

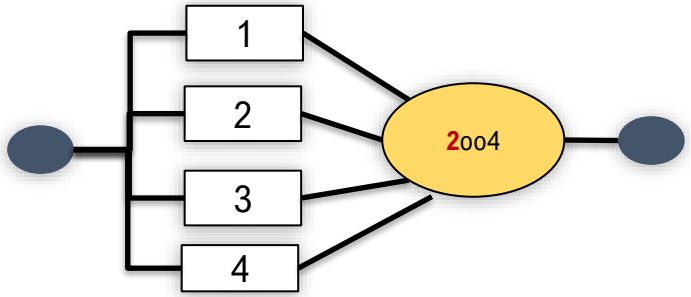



## Variant 2

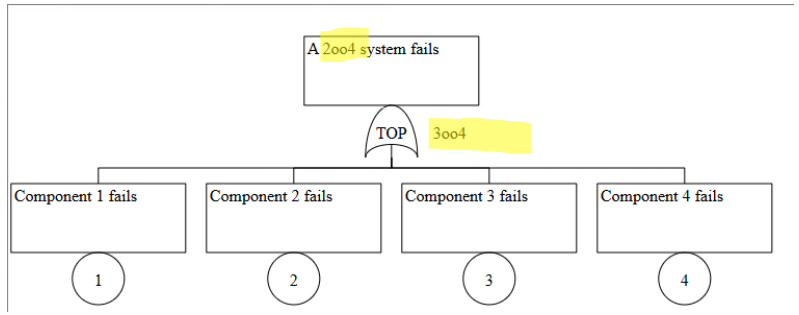
A 2oo3 voted system for functioning is also 2oo3 with respect to failure (symmetric)



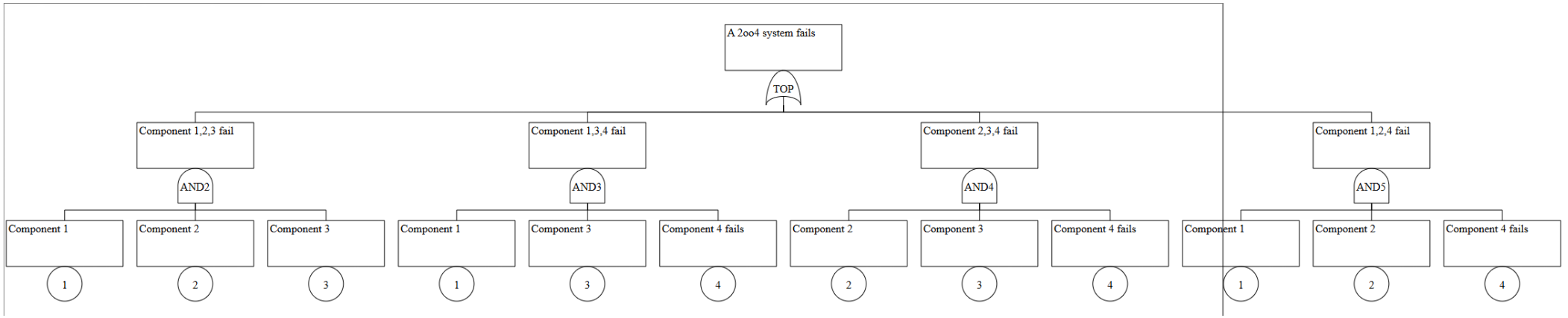
# RBD (reliability block diagram) vs FT



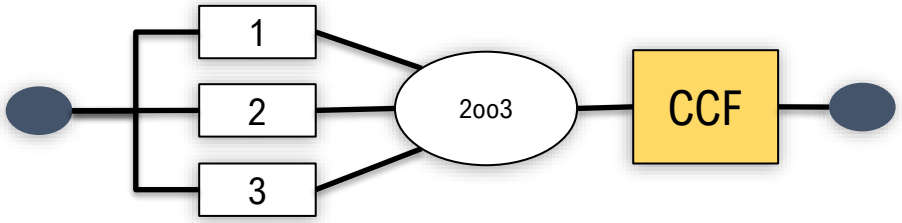

Variant 2



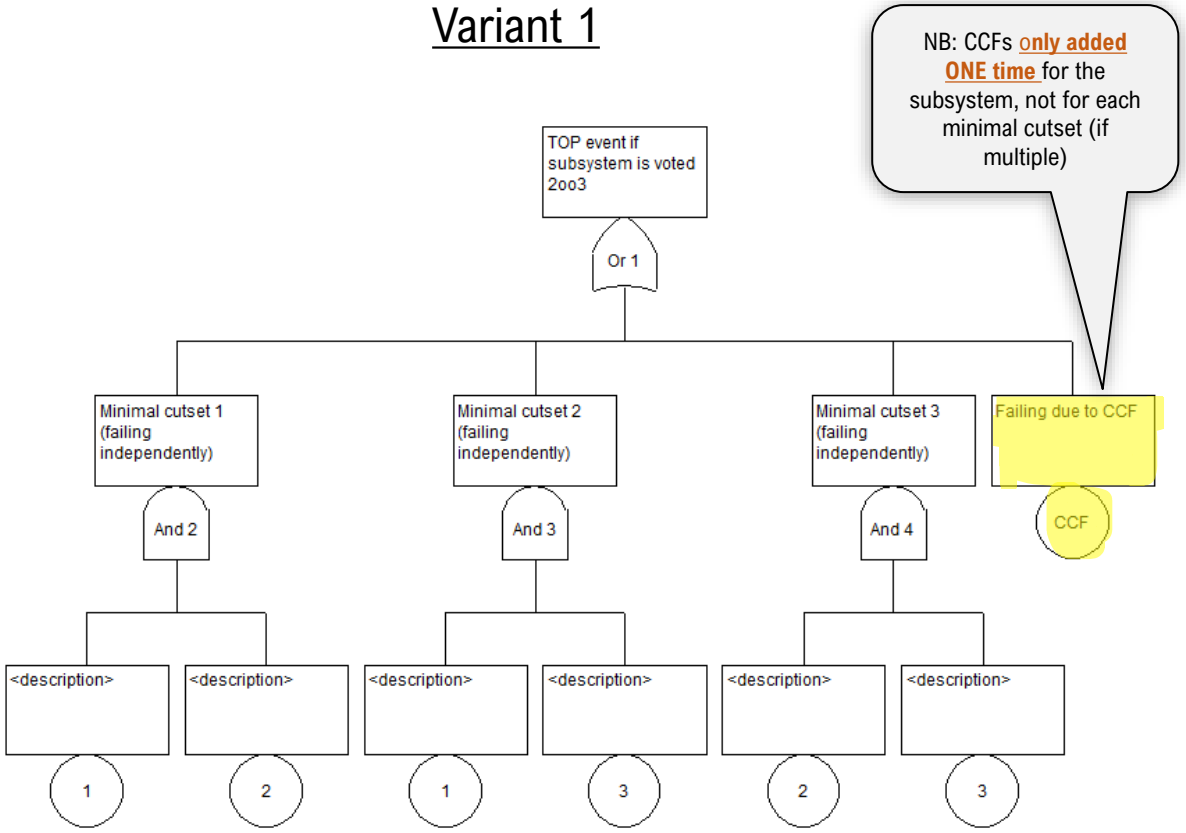

Variant 1



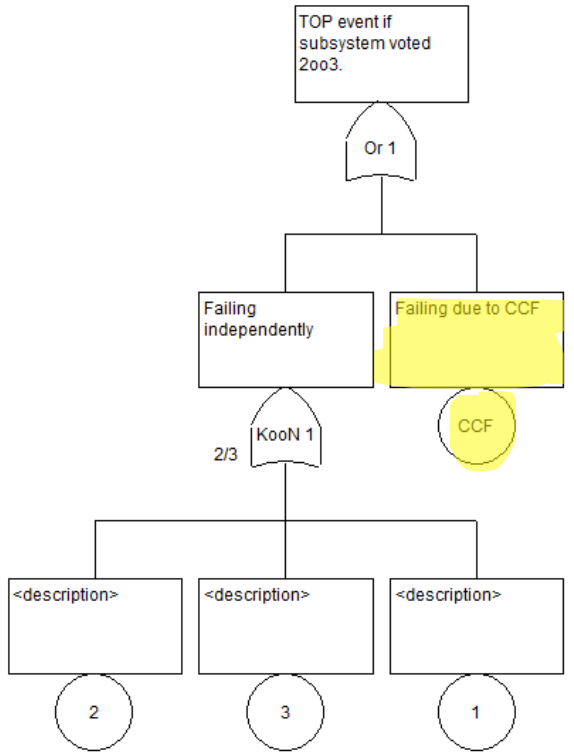
# Inclusion of CCFs




## Variant 1




## Variant 2

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# Miniprosjekt: Oppgave 3

Feiltreanalyse er et populært alternativ til pålitelighetsblokkskjema. Noen synes feiltre er enklere å forstå rent visuelt enn et blokkskjema, men det er nok litt «smak og behag». Alt som modelleres i et pålitelighetsblokkskjema kan også uttrykkes i et feiltre.

a) Lag to feiltrær i CARA_FT for TOPP-hendelse «SIF svikt» (i betydning at SIF ikke er i stand til å respondere «on demand»):

   i. Ett feiltre der kun uavhengige DU feil hensyntas

   ii. Ett feiltre der også fellesfeil (CCF) er med (standard beta faktormetoden).

b) For hvert av de to feiltrærne:

   i. Hent ut de minimale kuttmengdene og forklar hva de uttrykker.

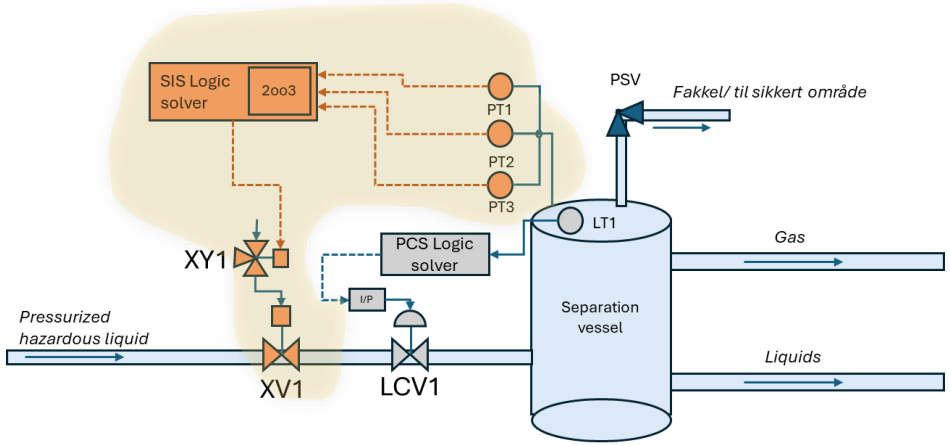
   ii. Legg inn pålitelighetsdata og regn ut PFD for hele SIFe, først uten fellesfeil (CCF) og deretter med fellesfeil (CCF) (standard beta faktormetoden). Kommenter kort på resultatet i forhold til oppgave 2 (RBD)

c) Sett opp formler og regn ut PFD for PT subsystem manuelt (uten bruk av CaraFT verktøy) basert på minimale kuttsett – med og uten CCFs. Forklar forskjellen på konservativ og ikke konservativ beregning. Sammenlign også hva CaraFT gir for akkurat dette subsystemet (bør da tegne opp PT systemet i en egen CaraFT fil).

d) Regn ut PFH for PT subsystem basert på minimale kuttsett (med og uten fellesfeil, standard beta faktormodellen)

e) Legg til resultatene fra oppgave 3c) og 3d) i tabellen fra oppgave 2e) slik at resultatene kan sammenlignes

f) Forklar hvordan feiltreet vil endres dersom voteringen for trykktransmitterne endres til 1oo3 eller 3oo4, både «variant 1» (der alle minimale kuttsett vises vises) og ved bruk av KooN port i feiltreet.

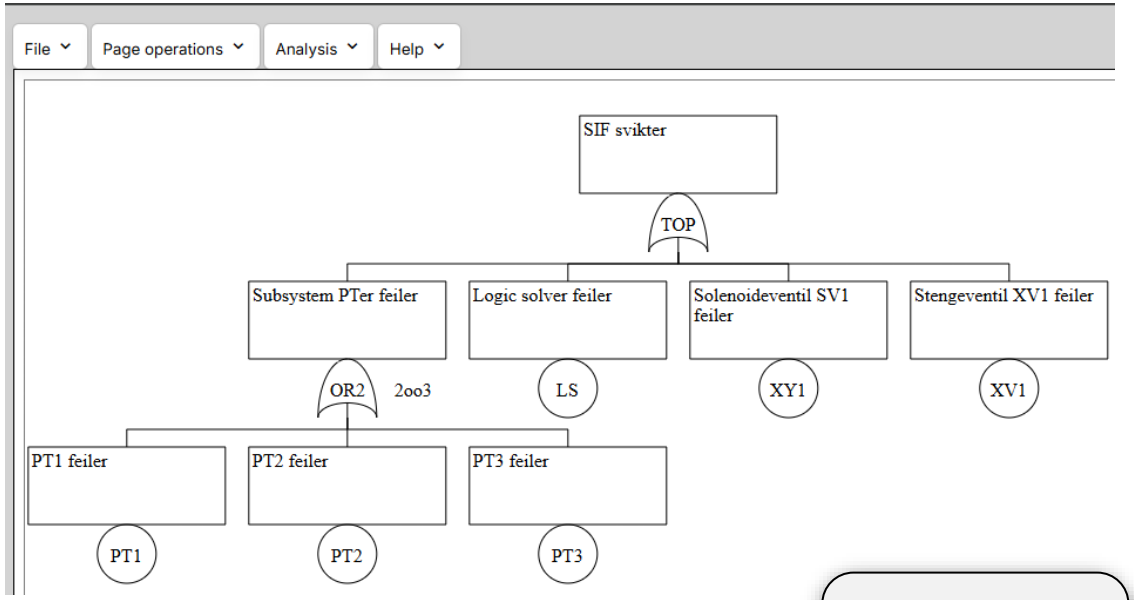



PSV: Pressure safety valve (purely mechanical, not a SIS)
PT: Pressure transmitter
I/P: Instrument (electrical) signal to pressure signal converter
LT: Level transmitter
LCV: Level control valve
XV: Shutdown valve
XY: Solenoid valve

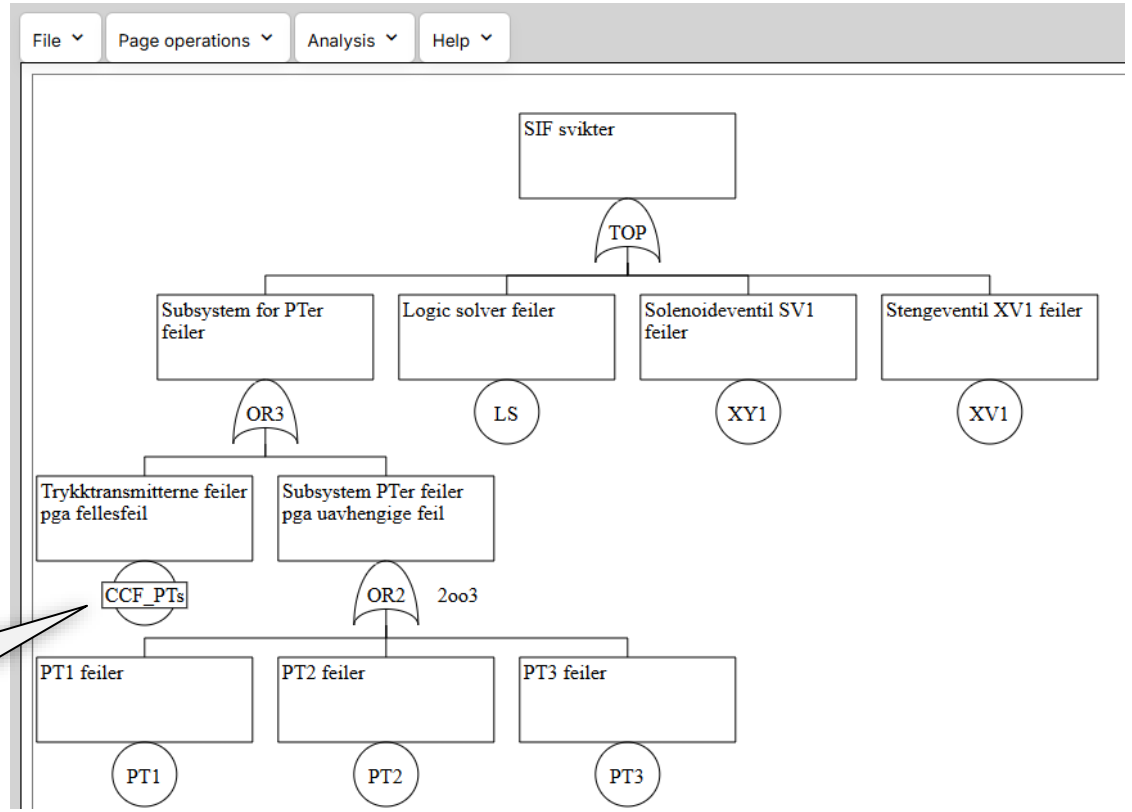
# Miniprosjekt oppgave 3: a) Feiltre SIF (Alternativ I)

Viser hvordan det gjøres i CaraFT i forelesningen!

## No CCFs




## With CCFs included

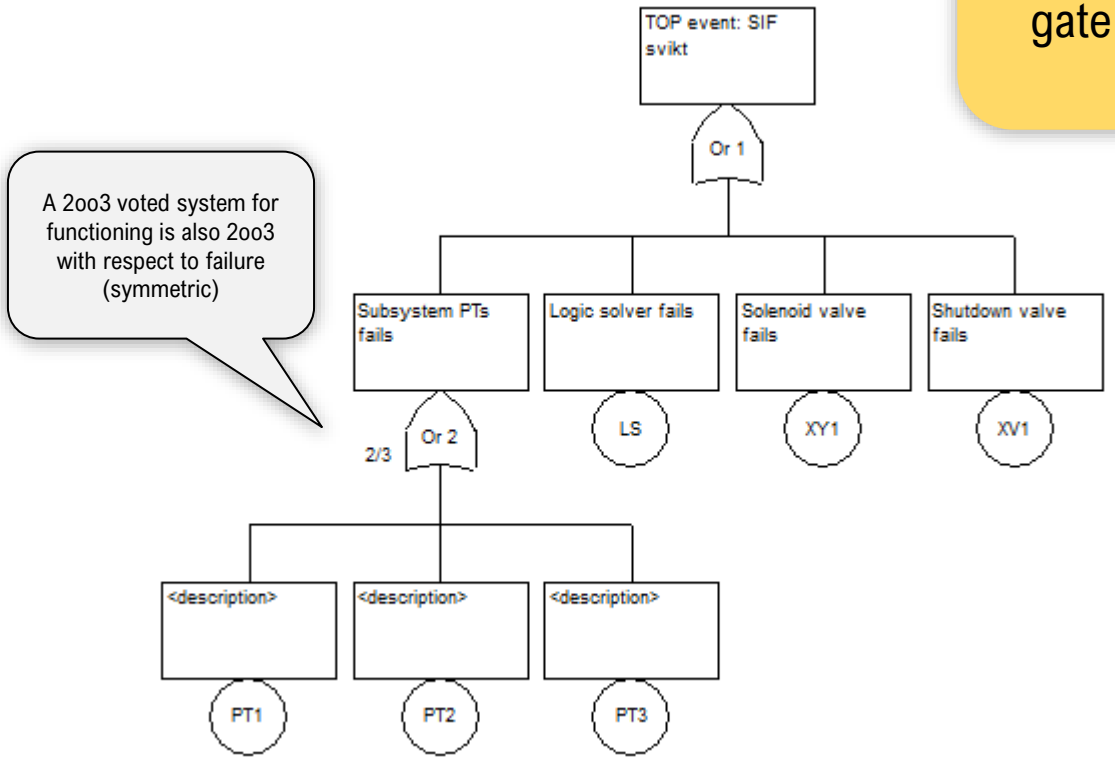



NB: CCFs **legges bare til en gang** for subsystemet (slik CCF legges til som en blokk i RBD)

# Miniprosjekt oppgave 3: a) Feiltre SIF (Alternativ II)

**Windows-based Excel CaraFT**

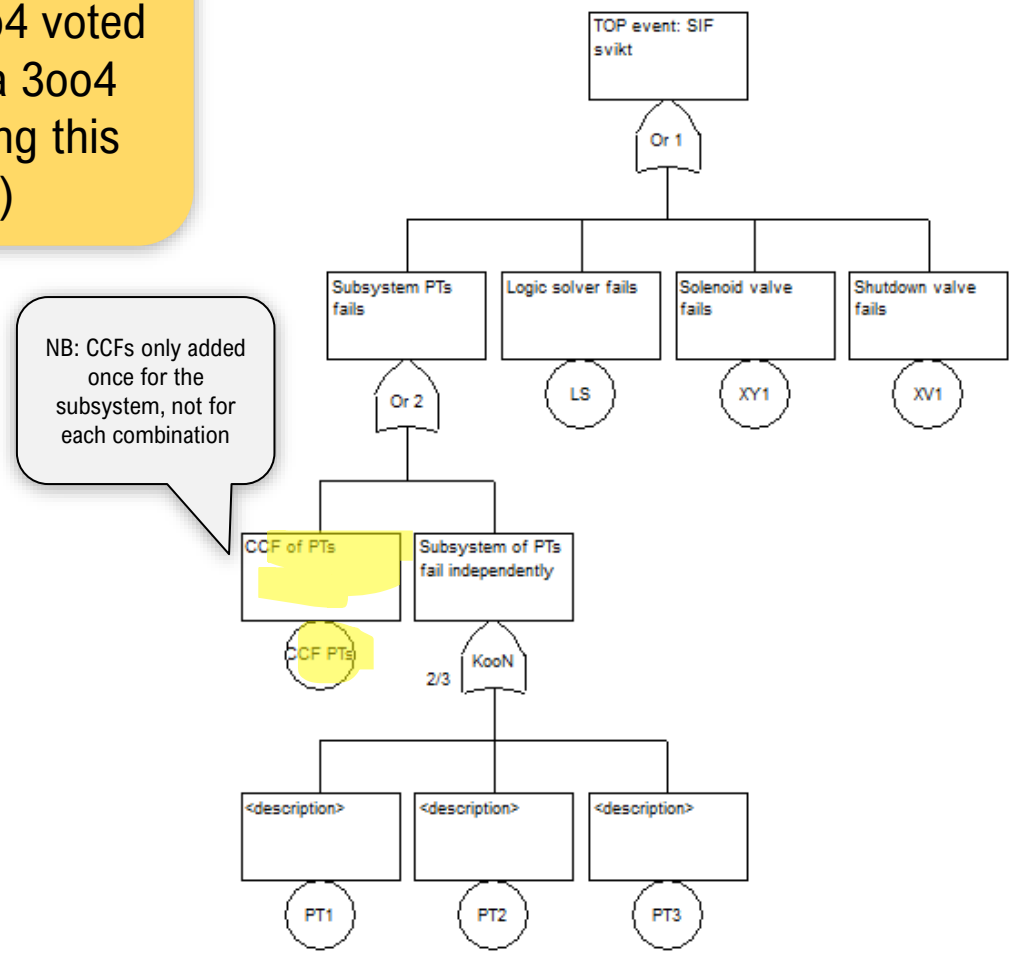
No CCFs




A 2oo3 voted system for functioning is also 2oo3 with respect to failure (symmetric)

**Discussion in plenum:** Why must a 2oo4 voted system apply a 3oo4 gate (if choosing this gate type)

With CCFs:

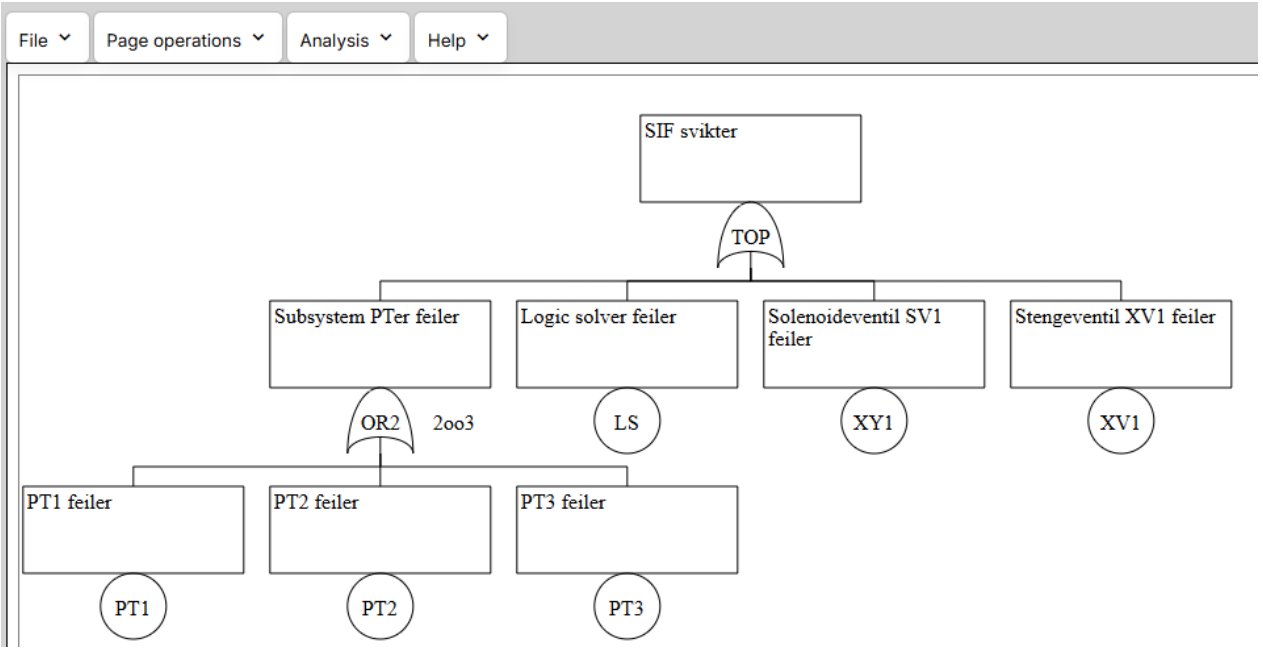



NB: CCFs only added once for the subsystem, not for each combination

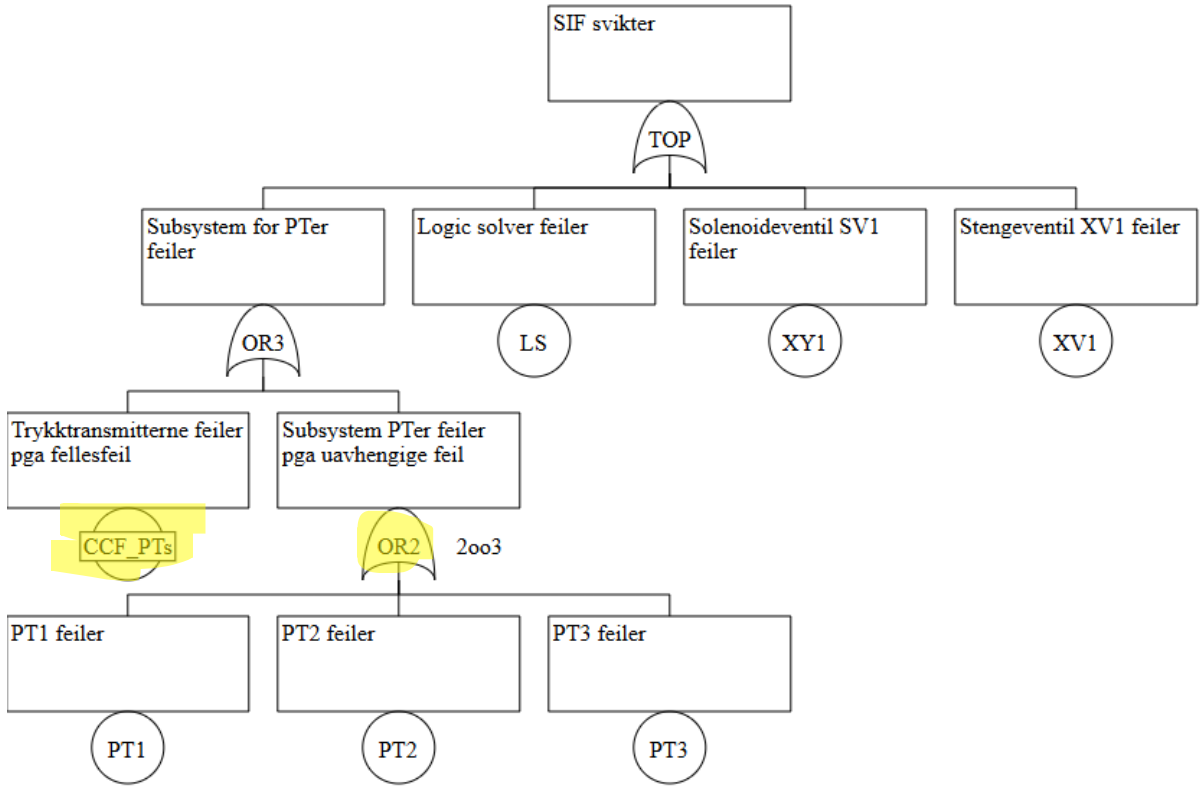
# Miniprosjekt oppgave 3: a) Feiltre SIF (Alternativ II)

**Web-basert CaraFT**

No CCFs




With CCFs:



# Miniprosjekt oppgave 3: b) – Minimale cutset m/CaraFT

**Windows-based Excel CaraFT**

No CCFs

Maximum cut size: 4 Mod. level: 0 Top event: Or 1

Cut set(s) with 1 component (Total: 3)
{LS}
{XY1}
{XV1}

Cut set(s) with 2 components (Total: 3)
{PT1,PT2}
{PT1,PT3}
{PT2,PT3}

Cut set(s) with 3 components (None found)

Cut set(s) with 4 components (None found)
**Total number of cut sets up to order 4: 6**

Her betyr cutset ***minimale*** cutset!

With CCFs

Maximum cut size: 4 Mod. level: 0 Top event: Or 1

Cut set(s) with 1 component (Total: 4)
{CCF_PT}
{LS}
{XY1}
{XV1}

Cut set(s) with 2 components (Total: 3)
{PT1,PT2}
{PT1,PT3}
{PT2,PT3}

Cut set(s) with 3 components (None found)

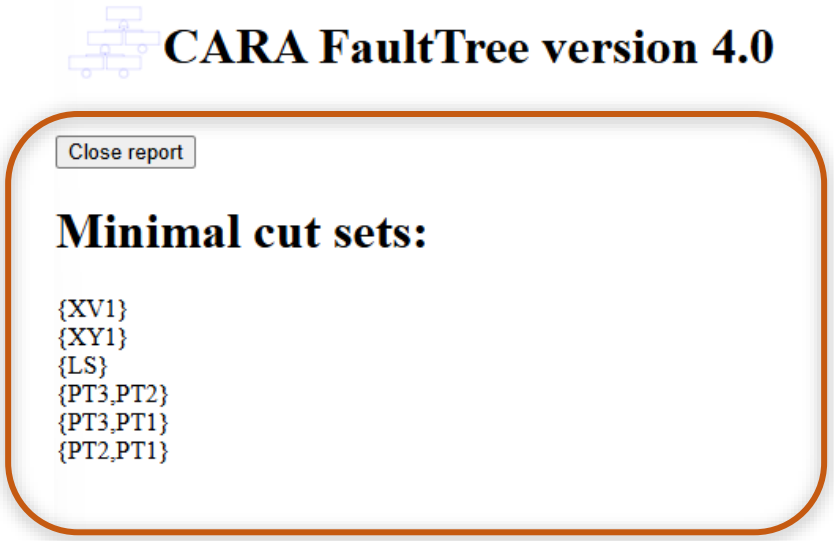
Cut set(s) with 4 components (None found)
**Total number of cut sets up to order 4: 7**

# Miniprosjekt oppgave 3: b) – Minimale cutset m/ CaraFT

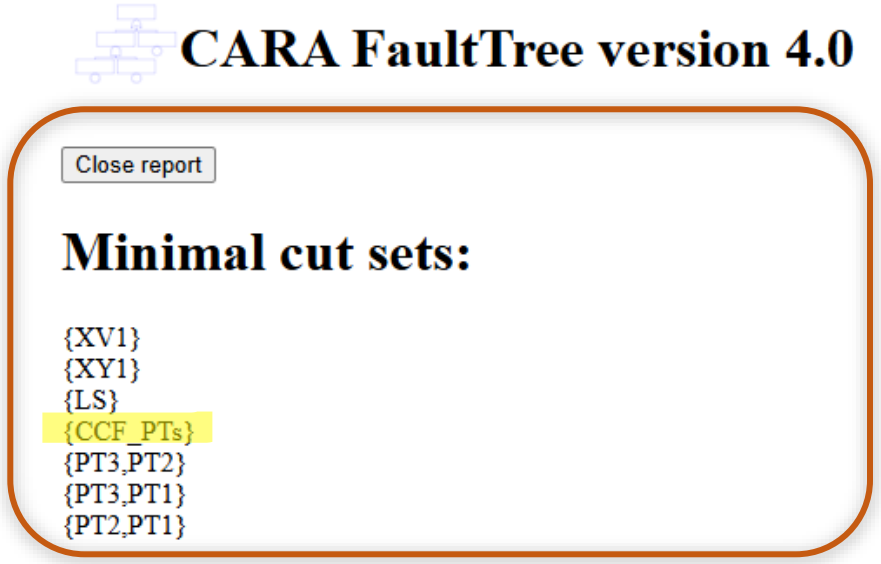
**Web-basert CaraFT**

Her betyr cutset ***minimale*** cutset!

No CCFs




With CCFs



# Quantitative analysis: What to quantify and notations

**What to quantify:**

- $Q_0(t)$: Pr(The TOP event occurs at time t)
- $Q_0$ : Average probability $Q_0$ of having the TOP event (“PFD”)
- Frequency of TOP event (time dependent or average) (“PFH”)

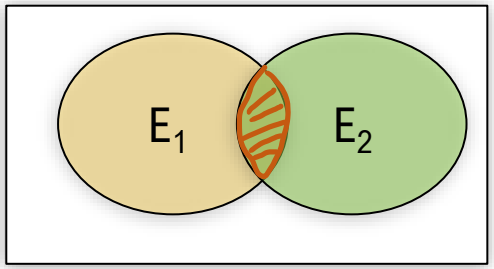
**Other notations:**

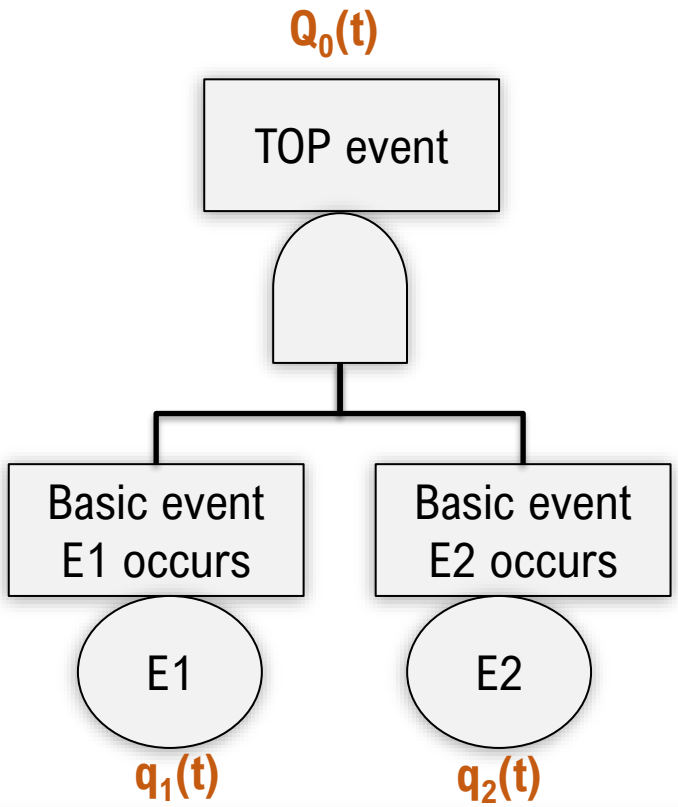
- $q_i(t)$: Pr(Basic event *i* occurs at time t)=Pr($E_i$)
- Qj(t):Pr(Minimal cut set *j* fails at time t)

# Input data data for basic events: Examples

| Assumptions | Equation | Remark |
|---|---|---|
| Exponentially distributed time to failure. **Non-repairable components.** | $q_i(t) = 1 - e^{-\lambda_i t} \approx \lambda_i t$ (when $\lambda_i t$ has small values) | |
| **Repairable components.** Occurs according to a Homogeneous Poisson process. | $q_i(t) \approx \lambda_i \cdot MTTR_i$ | Failures rate to cover evident failures only. |
| Repairable and periodically tested. | $q_{i,avg} \approx \frac{\lambda_i \cdot \tau_i}{2}$  | Failure rate to cover hidden failures only |

# Quantification with AND gate: TOP event probability $Q_0(t)$




If E1 and E2 are independent:

$$Q_0(t) = \Pr(E_1(t) \cap E_2(t)) = \Pr(E_1(t)) \cdot \Pr(E_2(t))$$

For the example:

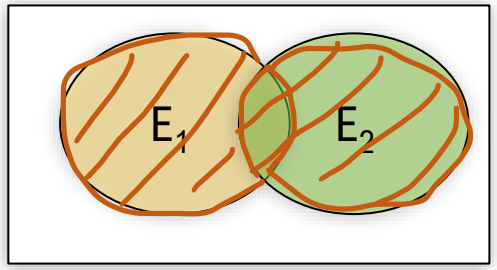
$$Q_0(t) = \prod_{i=1}^{2} q_i(t) = q_1(t) \cdot q_2(t)$$

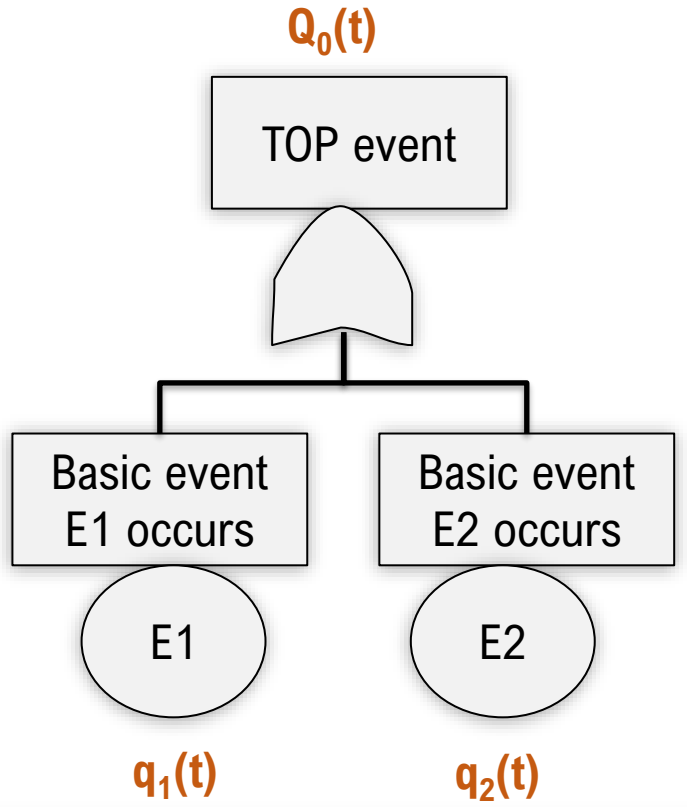
General formula for AND gate:

$$\boxed{Q_0(t) = \prod_{i=1}^{m} q_i(t)}$$

Where *m* is the number of basic events connected to the AND gate.

# Quantification with OR gate: TOP event probability $Q_0(t)$




If E1 and E2 are independent:

$$Q_0(t) = \Pr(E_1(t) \cup E_2(t)) = \Pr(E_1(t)) + \Pr(E_2(t)) - \Pr(E_1(t) \cap E_2(t))$$

For the example:

$$Q_0(t) == q_1(t) + q_2(t) - q_1(t) \cdot q_2(t)$$

$$= 1 - (1 - q_1(t)(1 - q_2(t)) = 1 - \prod_{i=1}^{2} (1 - q_i(t))$$

General formula for OR gate:

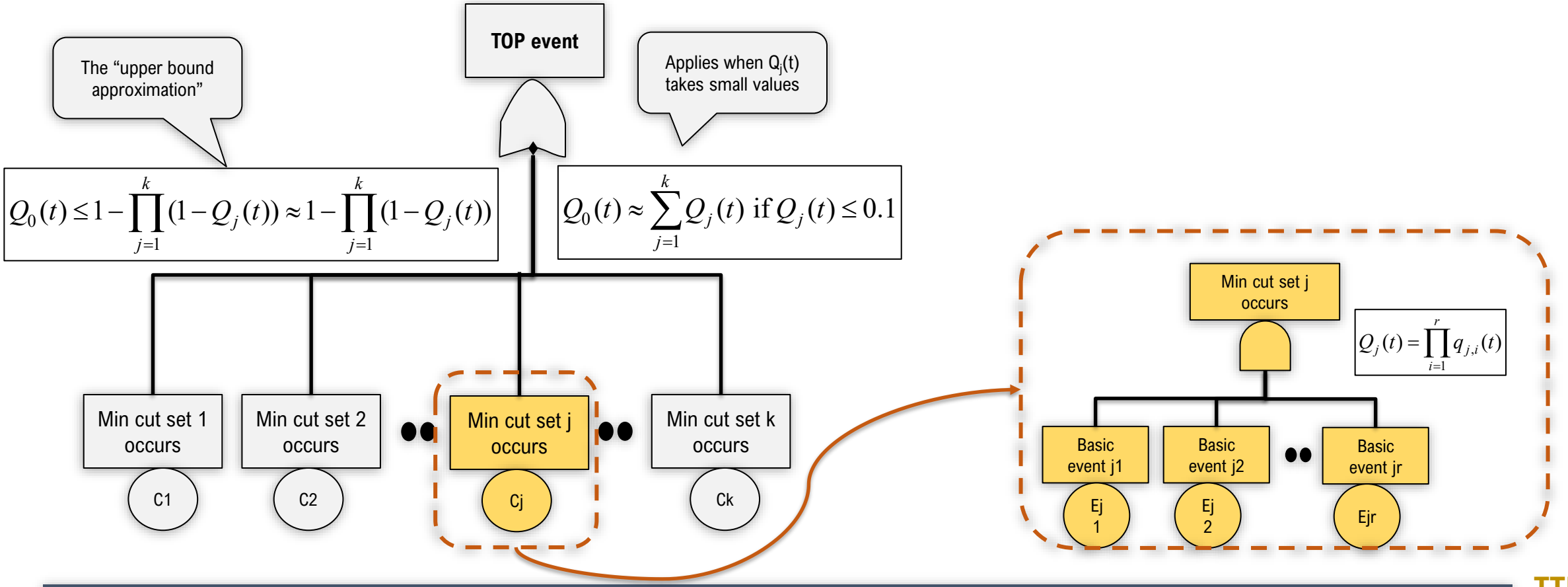
$$\boxed{Q_0(t) = 1 - \prod_{i=1}^{m} (1 - q_i(t))}$$

Where *m* is the number of basic events connected to the OR gate.
For small values of $q_i(t)$:

$$Q_0(t) \approx \sum_{i=1}^{m} q_i(t)$$

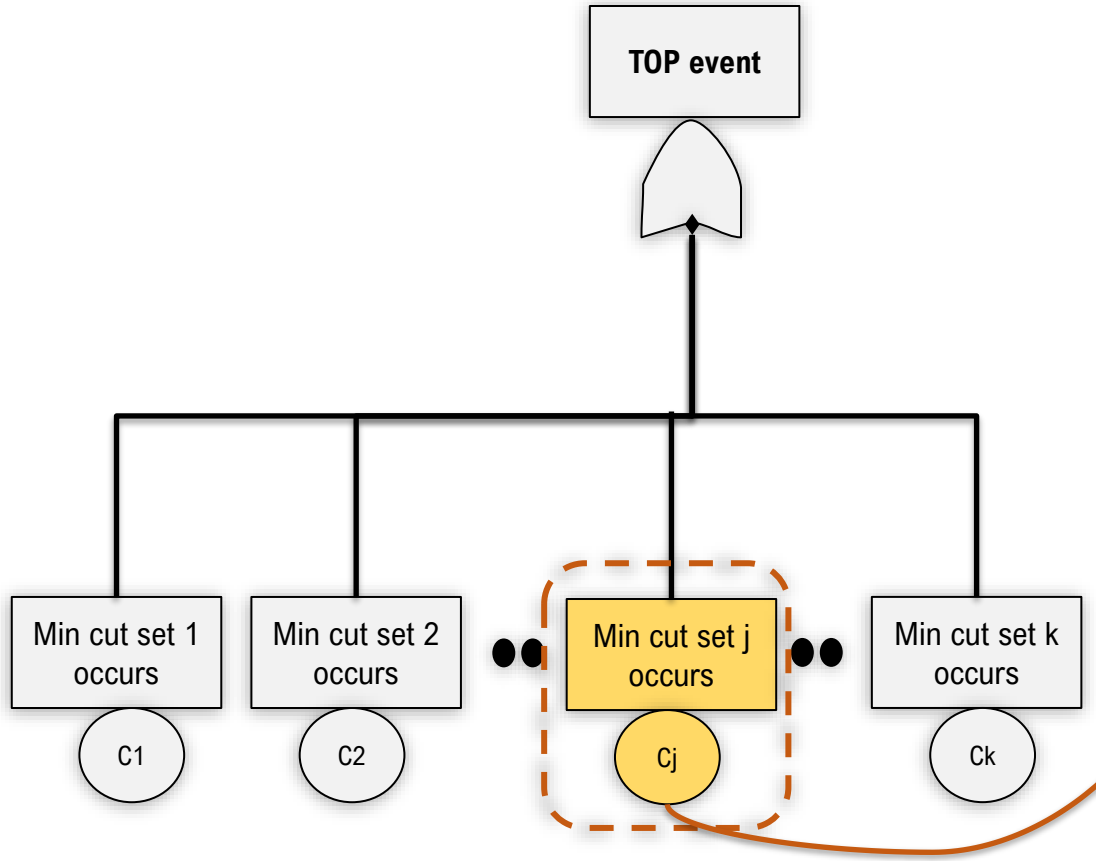
# $Q_0(t)$ based on minimal cut sets

- Fault tree may be reorganized according to the minimal cutsets
- TOP event occurs if one or more of the minimal cutsets occur



# *Average* $Q_0$ as PFD based on minimal cut sets

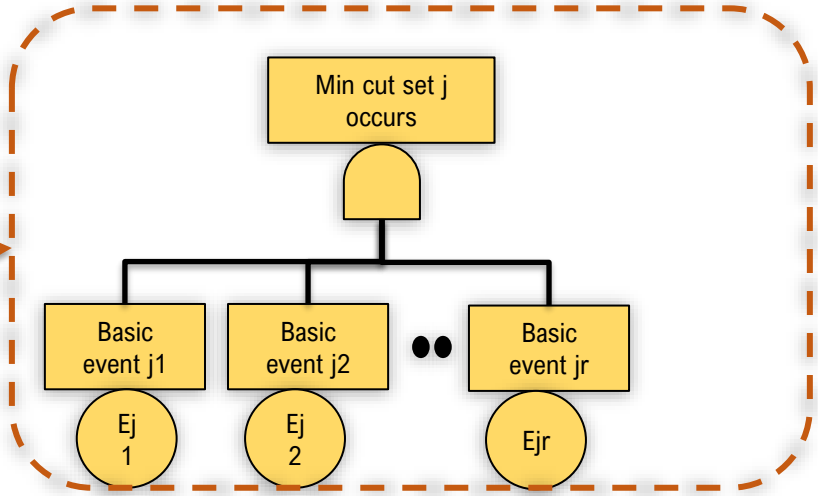
$$PFD_{Total} = Q_0 = 1 - \prod_{j=1}^{k}(1 - Q_j) \approx \sum_{j=1}^{k} Q_j \text{ if } Q_j \le 0.1$$




Replace by

**Conservative approach**

$$PFD_j = Q_j = \frac{1}{\tau}\int_0^{\tau}\prod_{i=1}^{r} q_{j,i}(t)dt$$




**Non conservative approach**

$$PFD_j = Q_j = \prod_{i=1}^{r}\left(\frac{1}{\tau}\int_0^{\tau} q_{j,i}(t)dt\right)$$

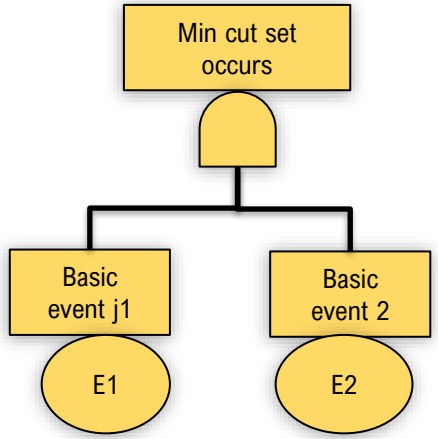
# *Average* $Q_0$ as PFD based on minimal cut sets: Example

**Conservative approach**

$$PFD_{MCS} = \frac{1}{\tau}\int_0^{\tau} q_1(t)\cdot q_2(t)dt$$

When $q_1(t)=q_2(t)=1\text{-}e^{-\lambda_{DU}t} \approx \lambda_{DU}t$, then

$$PFD_{MCS} = \frac{1}{\tau}\int_0^{\tau} (\lambda_{DU}t)^2 dt = \frac{(\lambda_{DU}\tau)^2}{3}$$




Minimal cutset: {E1,E2}

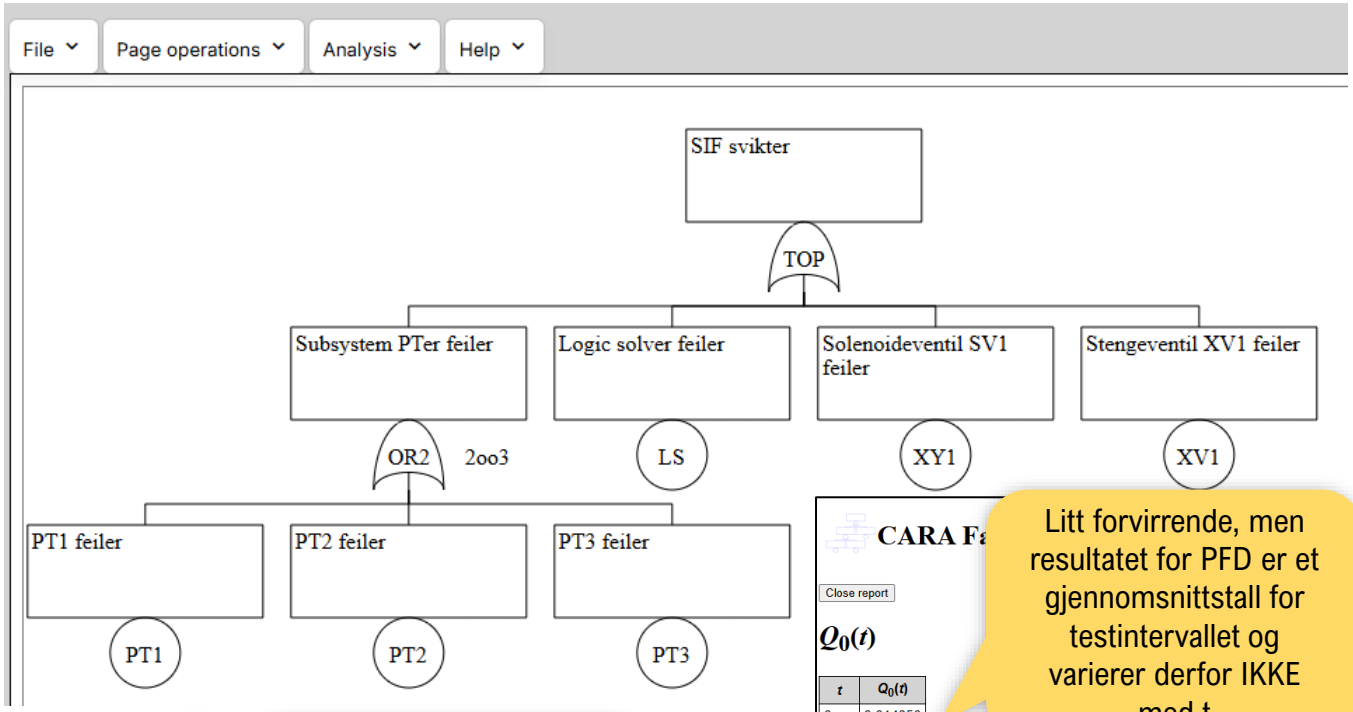
**Non conservative approach**

$$PFD_{MCS} = \prod_{i=1}^{2}\left(\frac{1}{\tau}\int_0^{\tau} q_i(t)dt\right)$$

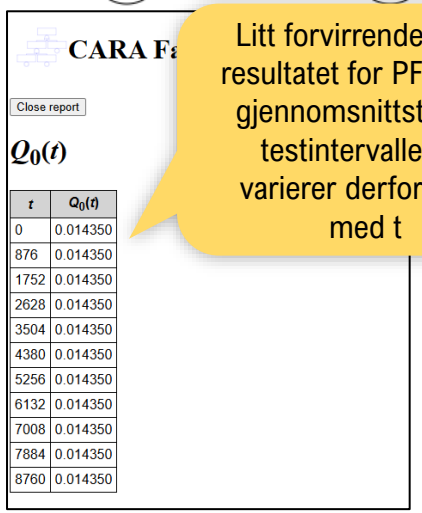
$$= \left(\frac{1}{\tau}\int_0^{\tau} \lambda_{DU}tdt\right)\cdot\left(\frac{1}{\tau}\int_0^{\tau} \lambda_{DU}tdt\right)$$

$$= \frac{\lambda_{DU}\tau}{2}\cdot\frac{\lambda_{DU}\tau}{2} = \frac{(\lambda_{DU}\tau)^2}{4}$$

# Miniprosjekt oppgave 3: b) – utregninger m/CaraFT



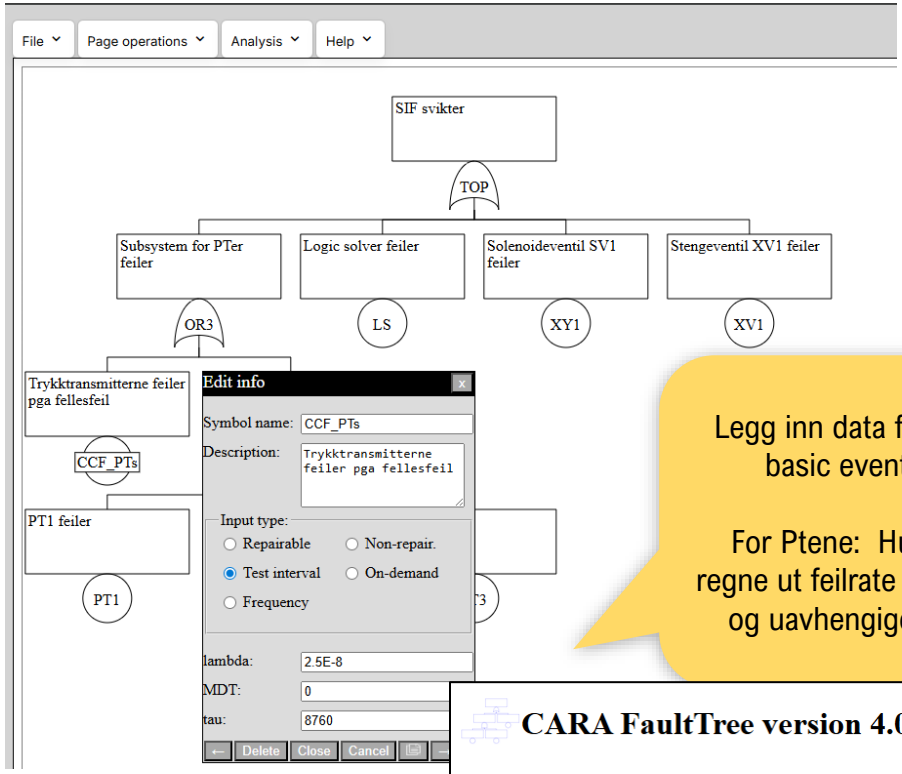

Legg inn data for alle basic events.




Litt forvirrende, men resultatet for PFD er et gjennomsnittstall for testintervallet og varierer derfor IKKE med t

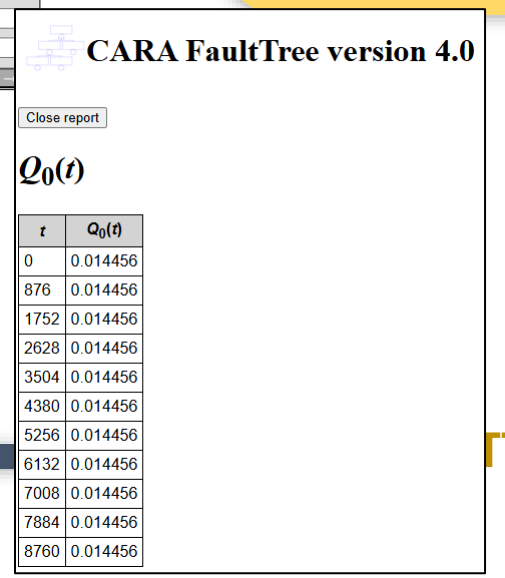
?

Hvorfor så liten forskjell i resultat med og uten fellesfeil?




Legg inn data for alle basic events.

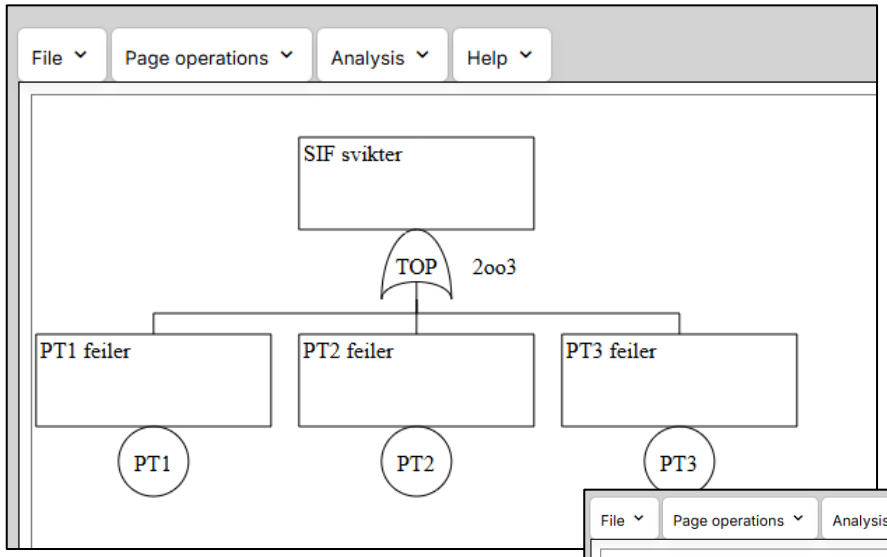
For Ptene: Husk å regne ut feilrate for CCF og uavhengige feil.

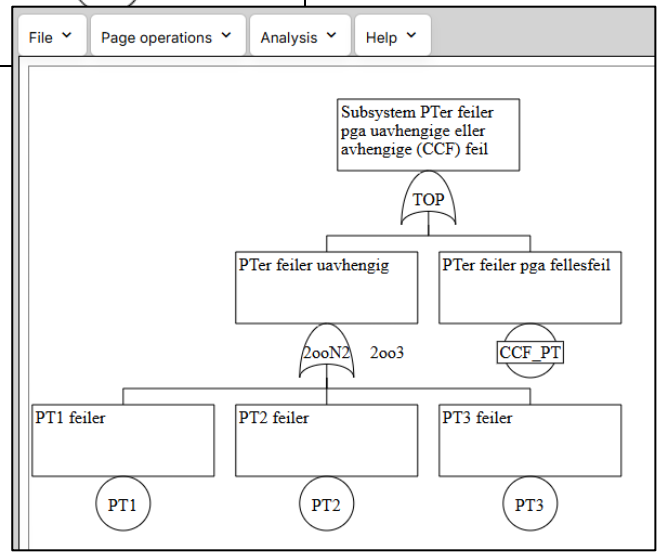


# Miniprosjekt oppgave 3: c) – Regne ut PFD for PT subsystem

*Tabell 1 Modellparametere som skal benyttes (feilrate er per time)*

| Pålitelighetsdata | *Parameter* | *Verdi* |
|---|---|---|
| DU feilrate trykktransmittere | $\lambda_{DU,PT}$ | 0.5E-6 per time |
| DU feilrate logic solver | $\lambda_{DU,LS}$ | 0,8E-6 per time |
| DU feilrate solenoid valve XY | $\lambda_{DU,XY}$ | 0,6E-6 per time |
| DU feilrate shutdown valve (XV) | $\lambda_{DU,XV}$ | 1,9E-6 per time |
| Fellesfeilandel trykktransmittere | β | 5% |
| Testintervall | τ | 8760 hours (1 år) |




**Minimal cutsets (uten fellesfeil):**
{PT1,PT2}, {PT2,PT3}, {PT1,PT3}

**Minimal cutsets (med fellesfeil):**
{PT1,PT2}, {PT2,PT3}, {PT1,PT3}, {CCF_PT}

# Miniprosjekt oppgave 3: c)

**Conservative approach**

Average of products

Each minimal cutset treates as an 1ooN voted system. I.e., 3 minimal cutsets where each is a 1oo2 voted system.

$$PFD_{PT} = 3 \cdot \left( \frac{\left(\lambda_{DU,PT}\tau\right)^2}{3} \right) = \left(\lambda_{DU,PT}\tau\right)^2$$

PFD for hvert minimalt cutsett

**Non-conservative approach**

Product of averages

$$Q_{PTs} = PFD_{PT} = 3 \cdot \left( \frac{\lambda_{DU,PT}\tau}{2} \cdot \frac{\lambda_{DU,PT}\tau}{2} \right) = \frac{3\left(\lambda_{DU,PT}\tau\right)^2}{4}$$

PFD for hvert minimalt cutsett

Some FT programs calculate Qj (for each minimal cutset) as **product of averages instead of the average of products**…

# Miniprosjekt oppgave 3: c) – Manuelt (konservativt)

Treating each MCS as 1ooN system. No CCFs

| Minimal cutsets PT subsystem | | | PFD per MCS |
|---|---|---|---|
| MCS1 | PT1 | PT2 | 6,39E-06 |
| MCS2 | PT1 | PT3 | 6,39E-06 |
| MCS3 | PT2 | PT3 | 6,39E-06 |
| | | | |
| | | | |
| | | PFD SIF (summed) | 1,92E-05 |
| | | PFD SIF (1-(1-...) etc) | 1,92E-05 |

Treating each MCS as 1ooN system. With CCFs

| Minimal cutsets PT subsystem | | PFD per MCS |
|---|---|---|
| PT1 | PT2 | 5,77E-06 |
| PT1 | PT3 | 5,77E-06 |
| PT2 | PT3 | 5,77E-06 |
| CCF_PT | | 1,10E-04 |
| | | |
| | PFD SIF (summed) | 1,27E-04 |

$$PFD_{PT,MCS1} = \frac{\left(\lambda_{DU,PT}\tau\right)^2}{3}$$

$$PFD_{PT,MCS2} = \frac{\left(\lambda_{DU,PT}\tau\right)^2}{3}$$

$$PFD_{PT,MCS3} = \frac{\left(\lambda_{DU,PT}\tau\right)^2}{3}$$

# Miniprosjekt oppgave 3: c) – Sammenligning m/ikke-konservativt

| Reliability data | Value | Notation | Independent | CCF |
|---|---|---|---|---|
| LDU, PT | 5,00E-07 | per hour | 4,75E-07 | 2,50E-08 |
| β_PTs | 5 % | | | |
| tau | 8760 | hours | | |

Konservativ

Ikke konservativ

Treating each MCS as 1ooN system. No CCFs

| Minimal cut set (MCS) | | | PFD per MCS |
|---|---|---|---|
| 4 | PT1 | PT2 | 6,39E-06 |
| 5 | PT1 | PT3 | 6,39E-06 |
| 6 | PT2 | PT3 | 6,39E-06 |
| | | | |
| | | | |
| | | PFD SIF | 1,92E-05 |

Non-conservative approach: Calculate PFD at basic event level. No CCFs

| Minimal cut set (MCS) | | | PFD per MCS |
|---|---|---|---|
| 4 | PT1 | PT2 | 4,80E-06 |
| 5 | PT1 | PT3 | 4,80E-06 |
| 6 | PT2 | PT3 | 4,80E-06 |
| | | | |
| | | | |
| | | PFD SIF | 1,44E-05 |

Treating each MCS as 1ooN system. With CCFs

| Minimal cut set (MCS) | | | PFD per MCS |
|---|---|---|---|
| 4 | PT1 | PT2 | 5,77E-06 |
| 5 | PT1 | PT3 | 5,77E-06 |
| 6 | PT2 | PT3 | 5,77E-06 |
| 7 | CCF_PT | | 1,10E-04 |
| | | | |
| | | PFD SIF | 1,27E-04 |

Non-conservative approach: Calculate PFD at basic event level. CCFs included

| Minimal cut set (MCS) | | | PFD per MCS |
|---|---|---|---|
| 4 | PT1 | PT2 | 4,33E-06 |
| 5 | PT1 | PT3 | 4,33E-06 |
| 6 | PT2 | PT3 | 4,33E-06 |
| 7 | CCF_PT | | 1,10E-04 |
| | | | |
| | | PFD SIF | 1,22E-04 |

# Miniprosjekt oppgave 3: c) – CaraFT

## CaraFT uten fellesfeil (CCF)

**CARA FaultTree version 4.0**

Close report

$Q_0(t)$

| $t$ | $Q_0(t)$ |
|---|---|
| 0 | 0.000014 |
| 876 | 0.000014 |
| 1752 | 0.000014 |
| 2628 | 0.000014 |
| 3504 | 0.000014 |
| 4380 | 0.000014 |
| 5256 | 0.000014 |
| 6132 | 0.000014 |
| 7008 | 0.000014 |
| 7884 | 0.000014 |
| 8760 | 0.000014 |

## CaraFT med fellesfeil (CCF)

**CARA FaultTree version 4.0**

Close report

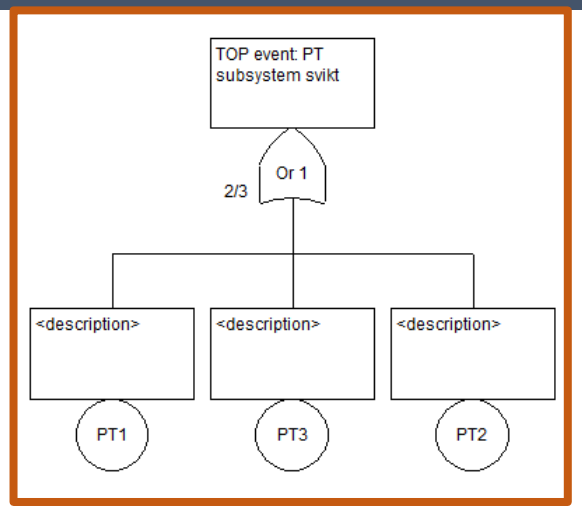
$Q_0(t)$

| $t$ | $Q_0(t)$ |
|---|---|
| 0 | 0.000122 |
| 876 | 0.000122 |
| 1752 | 0.000122 |
| 2628 | 0.000122 |
| 3504 | 0.000122 |
| 4380 | 0.000122 |
| 5256 | 0.000122 |
| 6132 | 0.000122 |
| 7008 | 0.000122 |
| 7884 | 0.000122 |
| 8760 | 0.000122 |

# Miniprosjekt oppgave 3: c) – Oppsummert om formler

## Konservativt (uten fellesfeil)

$$PFD_{PT} = 3 \cdot \left( \frac{\left(\lambda_{DU,PT}\tau\right)^2}{3} \right) = \left(\lambda_{DU,PT}\tau\right)^2$$



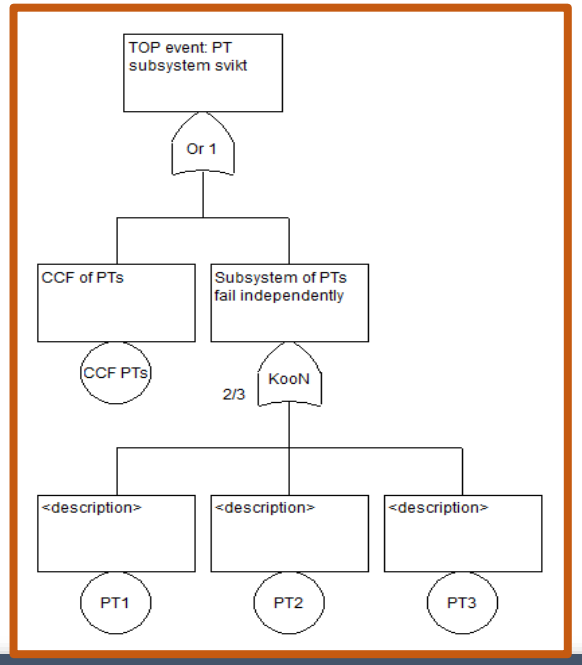

## Ikke-konservativ (uten fellesfeil)

$$PFD_{PT} = 3 \cdot \left( \frac{\lambda_{DU,PT}\tau}{2} \cdot \frac{\lambda_{DU,PT}\tau}{2} \right) = \frac{3\left(\lambda_{DU,PT}\tau\right)^2}{4}$$

## Konservativt (med fellesfeil)

$$PFD_{PT} \approx 3 \cdot \left( \frac{\left((1-\beta)\lambda_{DU,PT}\tau\right)^2}{3} \right) + \left( \frac{\beta\lambda_{DU,PT}\tau}{2} \right)$$

$$= \left((1-\beta)\lambda_{DU,PT}\tau\right)^2 + \frac{\beta\lambda_{DU,PT}\tau}{2}$$

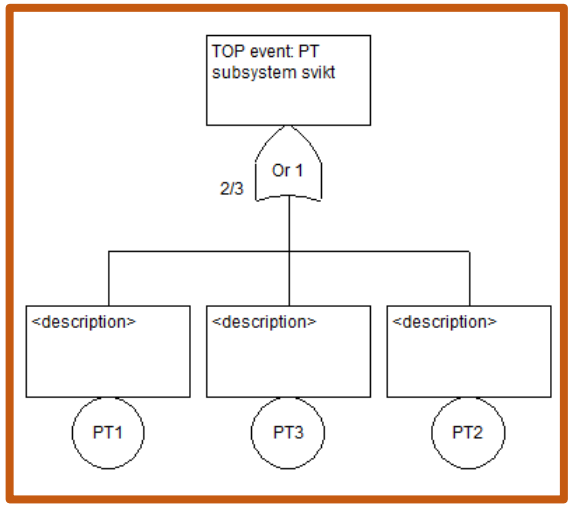


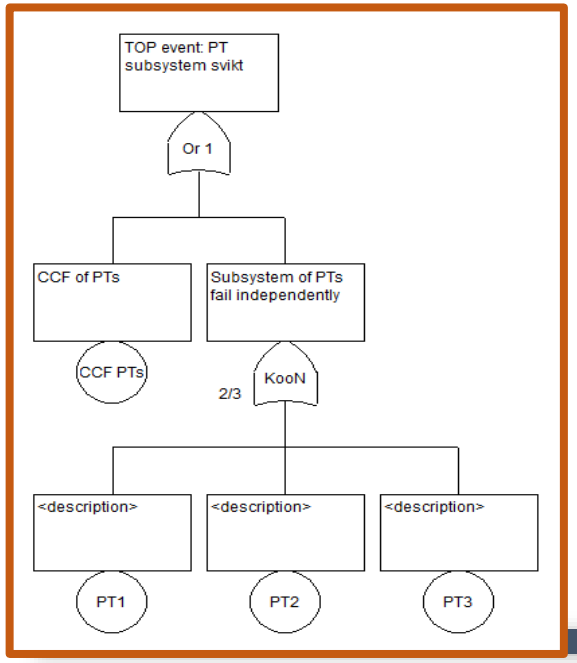

## Ikke-konservativ (med fellesfeil)

$$PFD_{PT} \approx 3 \cdot \left( \frac{(1-\beta)\lambda_{DU,PT}\tau}{2} \cdot \frac{(1-\beta)\lambda_{DU,PT}\tau}{2} \right) + \left( \frac{\beta\lambda_{DU,PT}\tau}{2} \right)$$

$$= \frac{3\left((1-\beta)\lambda_{DU,PT}\tau\right)^2}{4} + \frac{\beta\lambda_{DU,PT}\tau}{2}$$

# Miniprosjekt oppgave 3: d) – Regne ut PFH for subsystem PT



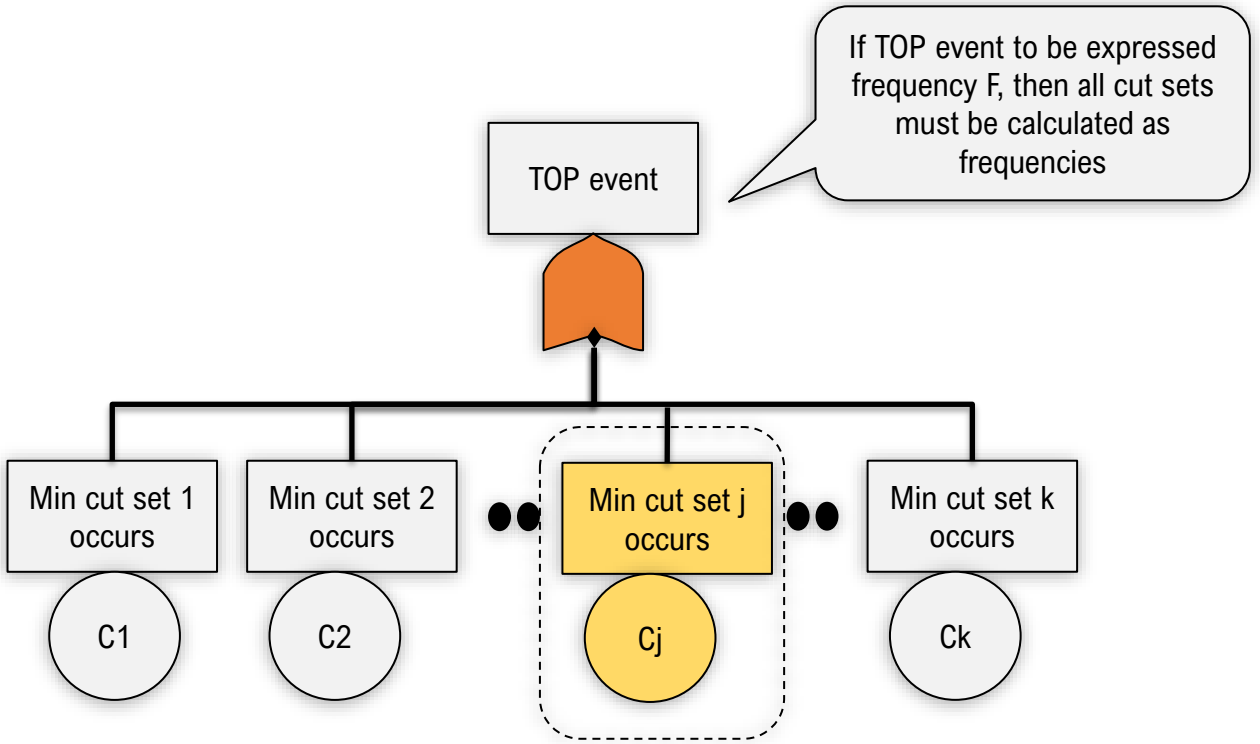

$$PFH_{PT} \approx \frac{3 \cdot \left(\lambda_{DU,PT}\tau \cdot \lambda_{DU,PT}\tau\right)}{\tau} = 3\lambda_{DU,PT}^{2}\tau$$

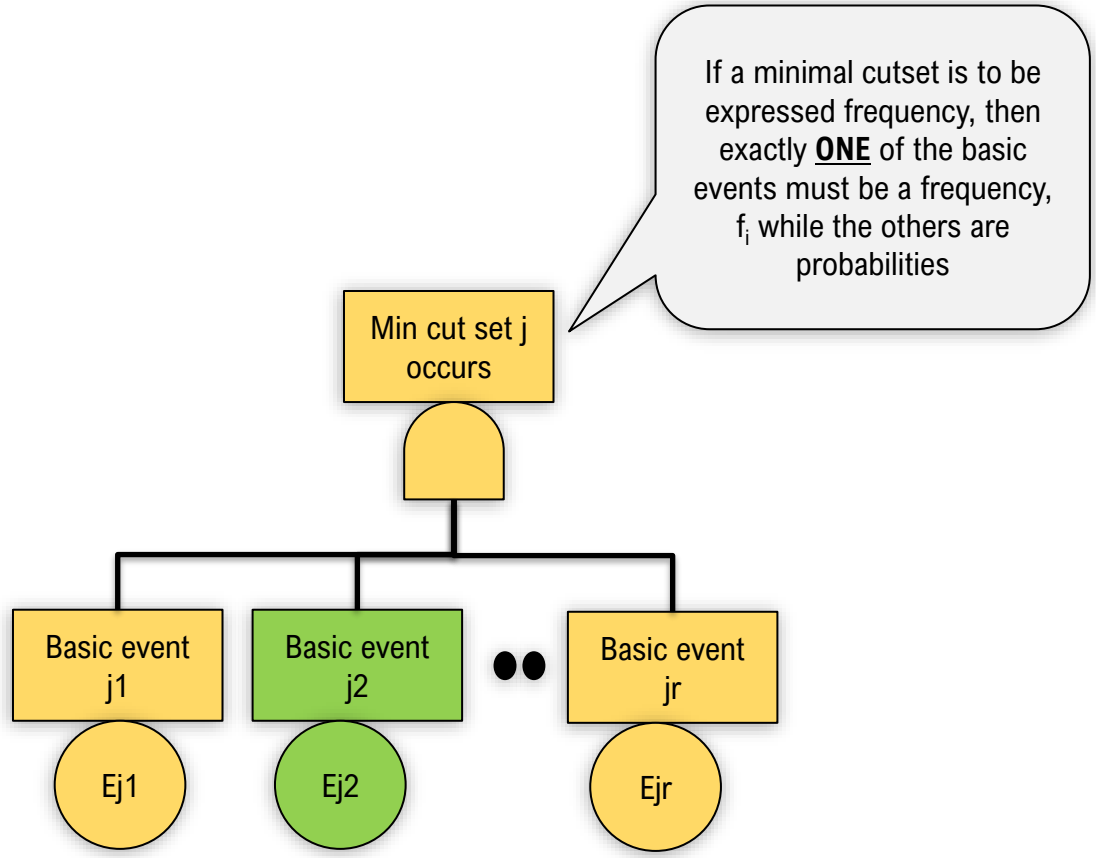
$$PFH_{PT} \approx 3 \cdot \left( \frac{\left((1-\beta)\lambda_{DU,PT}\tau\right)^{2}}{\tau} \right) + \beta_{PT}\lambda_{DU,PT}$$

$$= 3\left((1-\beta)\lambda_{DU,PT}\right)^{2}\tau + \beta_{PT}\lambda_{DU,PT}$$

| Reliability data | Value | |
|---|---|---|
| LDU, PT | Reference to SIF worksheet | |
| β_PTs | | |
| tau | | |
| | | |
| Treating each MCS as 1ooN system. No CCFs | | |
| **Minimal cutsets PT subsystem** | | **PFH per MCS (per hour)** |
| PT1 | PT2 | 2,19E-09 |
| PT1 | PT3 | 2,19E-09 |
| PT2 | PT3 | 2,19E-09 |
| | | |
| | | |
| | PFD SIF (summed) | 6,57E-09 |
| | PFD SIF (1-(1-...) etc) | |
| | | |
| Treating each MCS as 1ooN system. With CCFs | | |
| **Minimal cutsets PT subsystem** | | **PFH per MCS (per hour)** |
| PT1 | PT2 | 1,98E-09 |
| PT1 | PT3 | 1,98E-09 |
| PT2 | PT3 | 1,98E-09 |
| CCF_PT | | 2,50E-08 |
| | | |
| | PFD SIF (summed) | 3,09E-08 |

# What if frequencies *and* probabilities are involved?




PFH formler er satt opp på denne måten faktisk

# Miniprosjekt oppgave 2: e)

KUN for Subsystem input elements (PTs)

| Result | Forenklede formler (oppgave 2) | Feiltreanalyse (oppgave 3) | Markov analyse (oppgave 4) |
|---|---|---|---|
| PFD | | | |
| PFD no CCFs | 1.92E-5 | 1.92E-5 (1.44E-5*/ 1.4E-5**) | |
| PFD w CCFs | 1.27E-4 | 1.27E-4 (1.22E-4*, 1.22**) | |
| PFD w CCF using PDS method | 2.38E-4 | | |
| PFH (per time) | | | |
| PFH no CCFs | 6.57E-9 | 6.57E-9 | |
| PFH w CFFs | 3.09E-8 | 3.09E-8 | |

*CARA FT windows, **Cara Ftweb. eller ved manuell utregning konservativt

# Importance measures

**Birnbaum importance measures $I^B(i|t)$:**

- Can express the importance of one specific basic event *i* in a fault tree
- Typically, it has a value between 0 and 1
- If large value (close to 1) → A small change in $q_i(t)$ value gives a large impact on the TOP event probability

$$I^B(i \mid t) = \frac{\partial Q_o(t)}{\partial q_i(t)}$$

Often calculated as

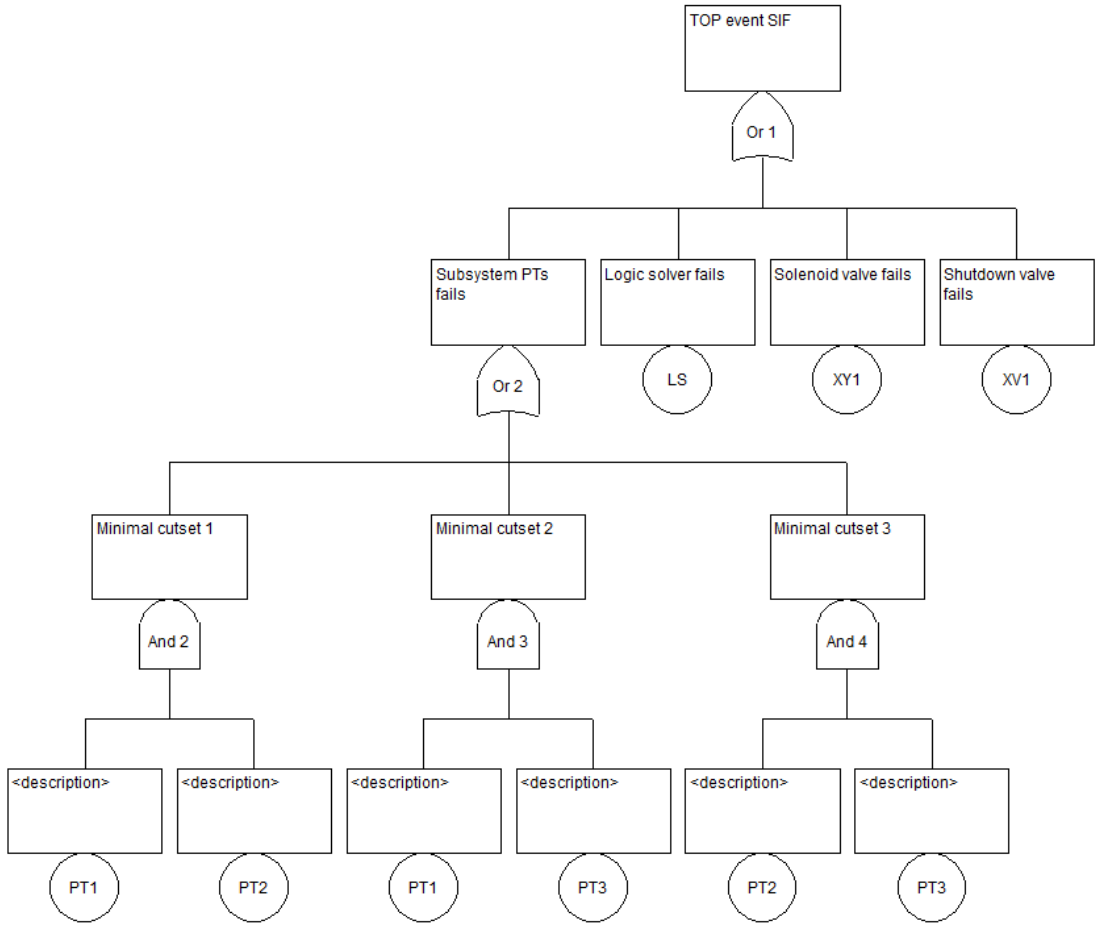
TOP event probability when basic event qi(t)=1

TOP event probability when basic event qi(t)=0

$$I^B(i \mid t) = \frac{Q_0(t \mid E_i) - Q_0(t \mid E_i^*)}{1-0} = Q_0(t \mid E_i) - Q_0(t \mid E_i^*)$$

If calculated for all basic events, it is possible to rank the importance of basic events relative to each other

# If time: Birnbaum measures for SIF




**CARA Fault Tree version 4.1 (c) Sydvest Software 1999**
**Licenced to: SINTEF Industrial Management**
**Dept of Safety and Reliability**
Date: 26.08.2024Time: 19:32:48

File: TTK2_SIF.CFT

New fault tree

Component importance

Method: Upper bound approximation

Maximum cut size: 4 Mod. level: 0 Top event: Or 1

Mission time t=8760

| Event | Low | High | Birnb. rel. |
|---|---|---|---|
| XV1 | 6,1243e-003 | 1,0000e+000 | 9,9388e-001 |
| LS | 1,0892e-002 | 1,0000e+000 | 9,8911e-001 |
| XY1 | 1,1757e-002 | 1,0000e+000 | 9,8824e-001 |
| PT3 | 1,4340e-002 | 1,8646e-002 | 4,3062e-003 |
| PT1 | 1,4340e-002 | 1,8646e-002 | 4,3062e-003 |
| PT2 | 1,4340e-002 | 1,8646e-002 | 4,3062e-003 |

TOP event probability if Ei* (=0)

TOP event probability if Ei* (=1)

# Avrunding

# Vi oppsummerer at…

- Feiltre er en mye brukt og intuitiv måte å avdekke hvordan et system feiler
- Minimale kuttsett er nyttige – kontrollere at feiltreet er rett. Rydder opp og gjør det enklere å regne på sannsynligheter og frekvenser for topphendelse
- For PFD regner CaraFT ut «ikke konservativt» (dvs man får litt lavere verdi)
- Det finner mål å regne på, som Birnbaum, for å identifisere hvor viktig enkelt-komponenter er