TTK2 – Pålitelighetsanalyse av sikkerhetsfunksjoner

Seminar 4 – Markov analysis

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https://www.itk.ntnu.no/emner/fordypning/TTK2

TTK2

Om bruk av innhold fra slidene

The content is updated regularly to improve precision and ensure relevance, which is reflected in the revision number. Please reach out to mary.a.lundteigen@ntnu.no if you have comments or suggestions for improvement.

La de gjerne inspirere, men husk å referere meg om du bruker noe fra presentasjonen direkte

Markov analysis in a nutshell

Markov analysis is an analysis of a **stochastic process** that describes the transitions between a **set of states**.

- Has many application areas, including reliability
- Unlike finite state machine, it has stochastic transitions rather than deterministic

Markov analysis relies on a **Markov model,** which is a graphical diagram that identifies states and the transition between them covering:

- Functioning states
- Degraded states
- Failed states
- Hazardous states
- …

With basis in the Markov model, we can **calculate** reliability measures, such as:

- Probability of functioning and/or failure (time dependent, steady state)
- Frequency of entering various states
- Mean time to failure

Figure: Russian mathematician Andrei Markov (1856-1922)

Markov model and Markov property

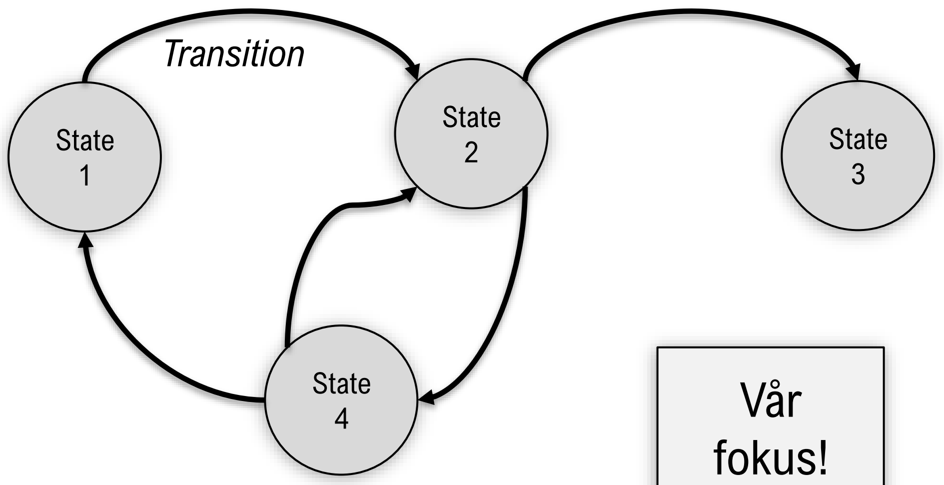
A Markov model includes:

- **States –** states of components, events/circumstances, …
- **Transitions:**
  - **Jumps** between states
  - Characterized by a **rate** (frequency)

Transitions must fulfil the **Markov property:**

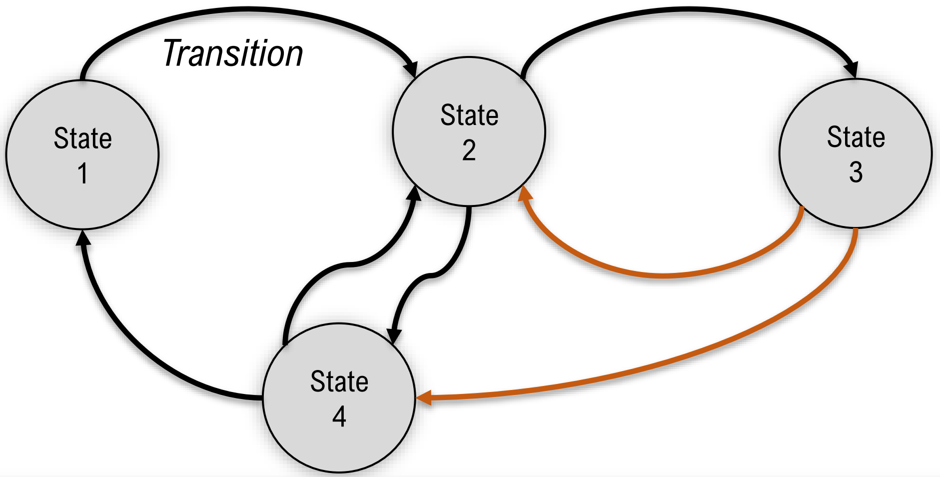
- Probability of leaving a state in the next time step is not dependent on what happened upfront (**memory less**)
- The consequence of Markov property is that all **transition rates are constant** (exponentially distributed time to jump).

Model with absorbing state for time dependent analysis



Vår fokus!

Model for steady state analysis



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Markov analysis step by step

1. Make a sketch of the system
2. Define the system states
3. Optional: Group similar states to one state or see if you can make other simplifications (reduce dimension)
4. Draw the Markov model with the transition rates
5. Perform quantitative assessment
6. Present (explain) the results

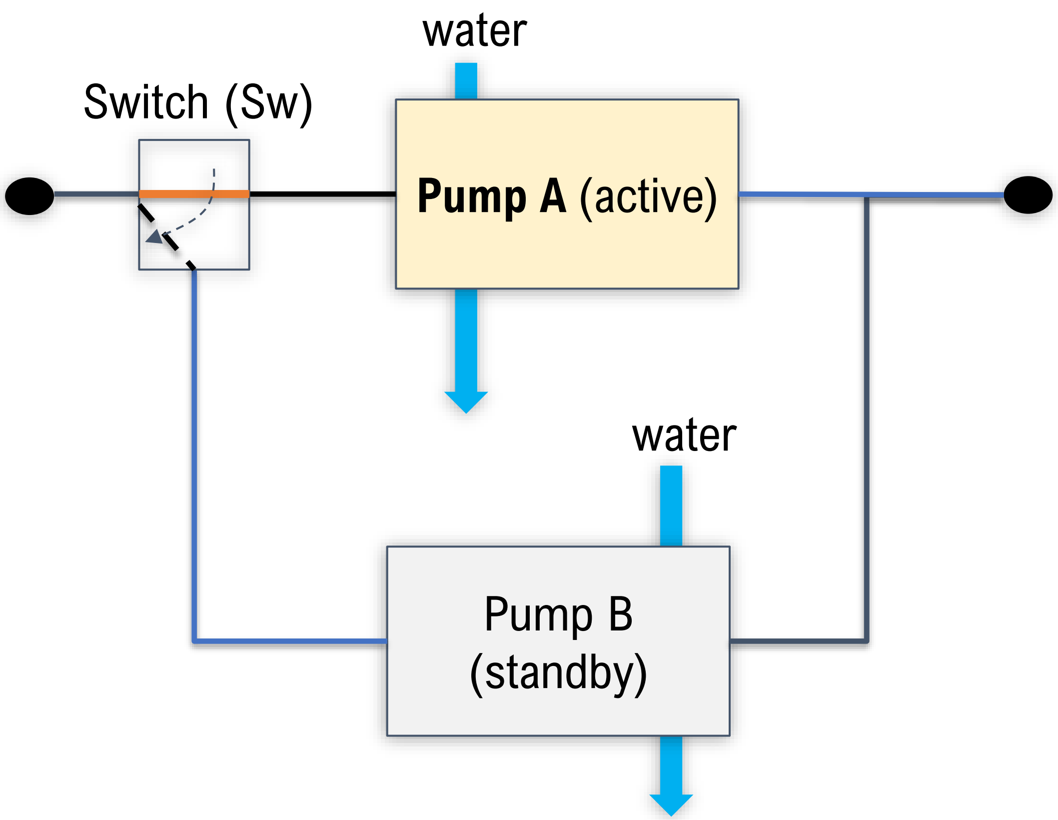
Example

A pump system consisting of:

- One active pump (pump A)
- One standby pump (pump B)
- One switch (Sw) that change from pump A to pump B upon pump A failure

System operation is characterized as follows:

- If pump A fails, the switch activates pump B so that it takes over.
- A repair is initiated when pump A has failed.
- If pump B fails before pump A is repaired, they must both be repaired before the system is put back into operation
- (We exclude states and transitions for repairing pump B while in standby)



Example: System states

- We define applicable states (x) for each component
- Each state is numbered
  - Zero is used for the functioning state (choice made)

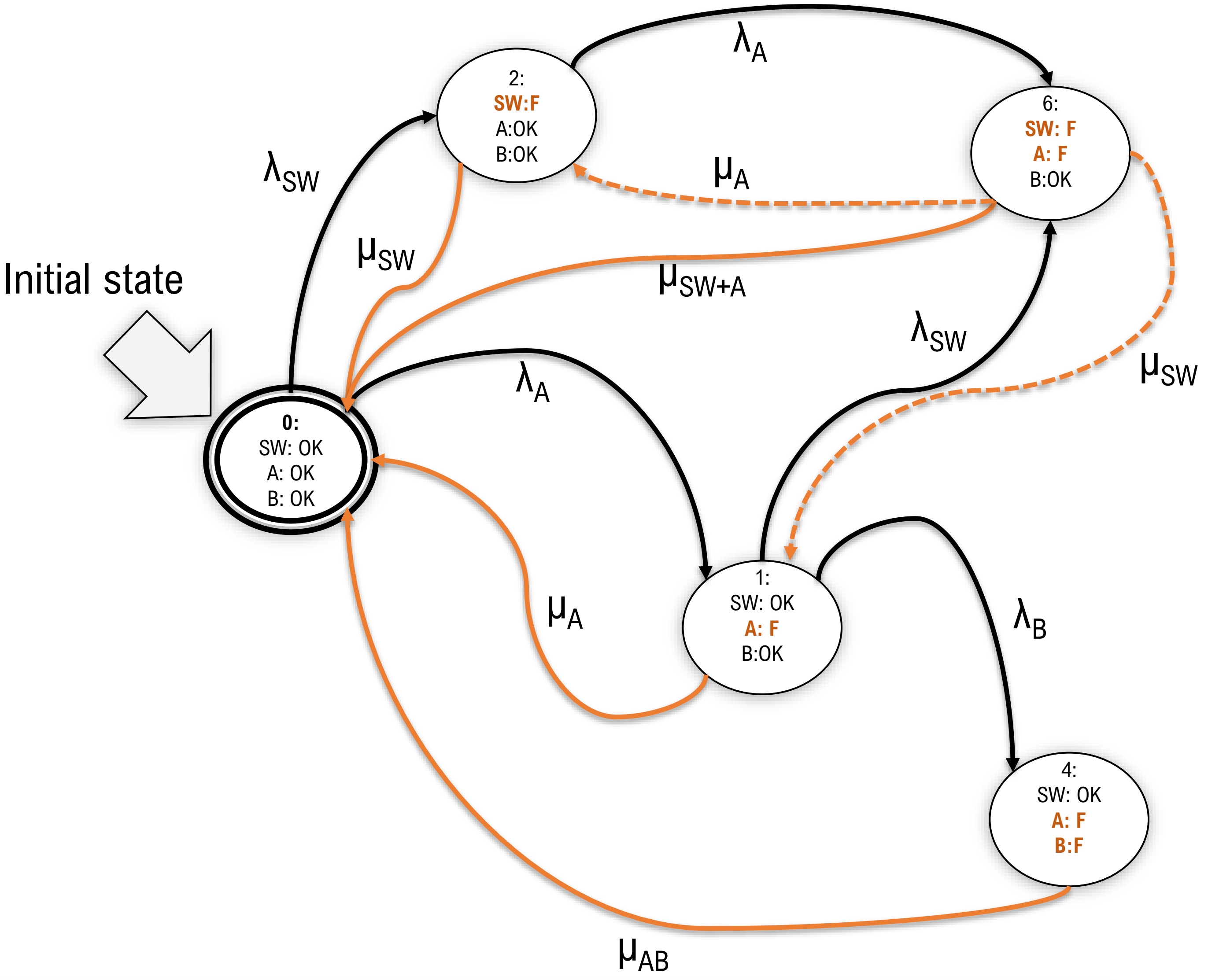
System state x_S	x_A	x_B	x_{Sw}	Comments
0	1	1	1	All components ok: Pump A active and running, pump B available to start, switch is functioning.
1	0	1	1	Pump A in fault state, switch functioning (by switching) and pump B available to start
2	1	1	0	Pump A is active and running, pump B is available in standby, but switch is in the failed state (not able to switch, if needed)
3	1	0	1	Pump B in fault state, pump A and switch ok ok
4	0	0	1	Pump A and pump B in fault state, switch ok
5	1	0	0	Pump B and switch in failed state, Pump A ok
6	0	1	0	Pump A has failed, pump B is available, but switch has failed and not able to reconnect to pump B.
7	0	0	0	All failed

Revised example: Markov model and state transition rates

The following transition rates are introduced:

- λ_A: Failure rate of active pump
- λ_B: Failure rate of standby pump
- λ_{SW}: Failure rate of switch

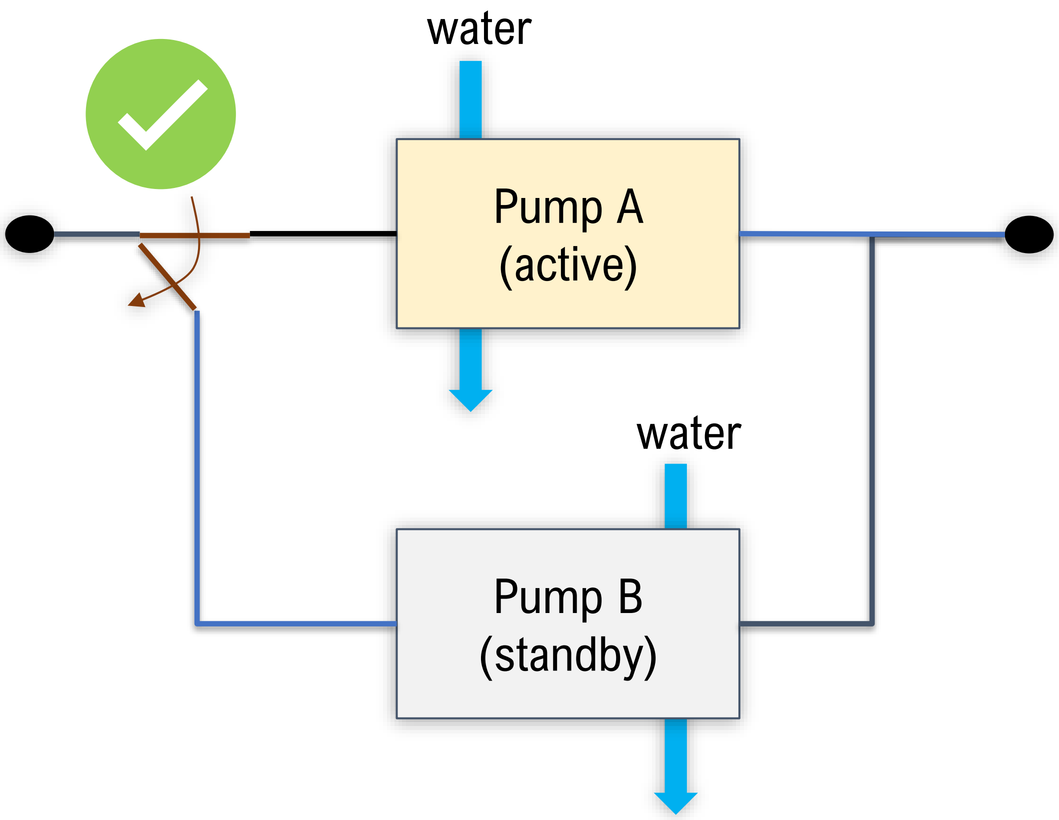
- μ_A: Repair rate of active pump A
- μ_{AB}: Repair rate of both pumps A and B
- μ_{SW}: Repair of switch
- μ_{SW+A}: Repair of switch and pump A and switch



Additional simplications

The number of states **may be reduced even further** with the following assumption:

- **Switch is perfect:** If pump A fails, pump B can always be started



Resulting states

- Definition of system states:

System state x_S	x_A	x_B	Comments
0	1	1	Both pumps are ok: Pump A is active and running, Pump B available in standby.
1	0	1	Active pump A is failed, Standby B has started and is running
2	0	0	Both pumps (A and B) and have failed while running

Using color codes to «rank» states

- Definition of system states:

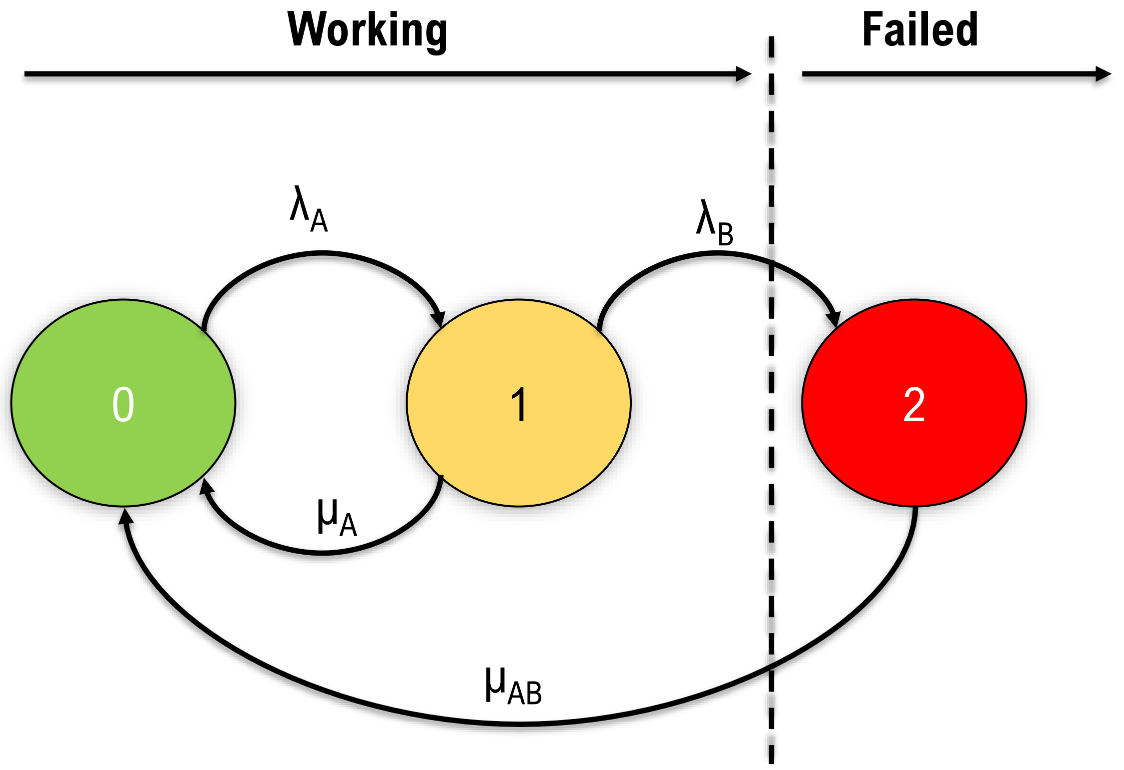
System state x_S	x_A	x_B	Comments
0	1	1	Both pumps are ok: Pump A is active and running, Pump B available in standby.
1	0	1	Active pump A is failed, Standby B has started and is running
2	0	0	Both pumps (A and B) and have failed while running

Revised example: Markov model and state transition rates

The following transition rates are introduced:

- λ_A: Failure rate of active pump
- λ_B: Failure rate of standby pump

- μ_A: Repair rate of active pump
- μ_{AB}: Repair rate of both pumps



- Pump A is active and running
- Pump B is available as standby

- Pump A has failed
- Pump B has started and is running

- Pump A still in failed state
- Pump B failed while running

Transition rate matrix

A **transition (rate) matrix**, here named **A**, identifies the rates (or alternatively probabilites) of **moving between** each state of the Markov model.

Procedure:

1. Fill inn all cells a_{ij}, **i i≠j** with rates in the Markov model

2. Fill inn the diagonal cells a_{ij}, **i=j** as follows:

$$a_{ii} = -\sum_{i \neq j} a_{ij}$$

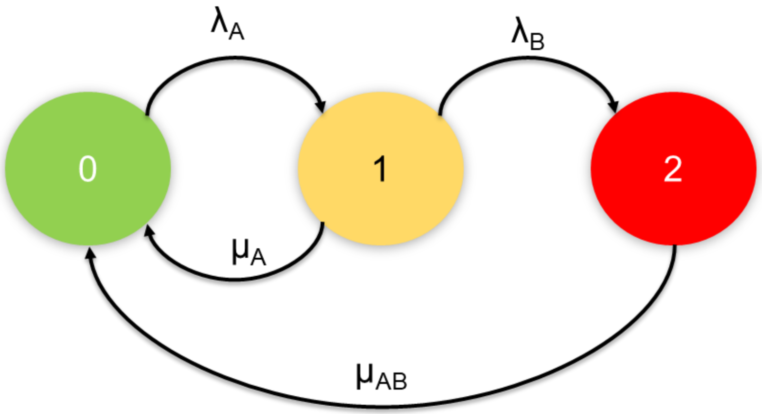
i.e., the total rate of staying **and** leaving must be zero. You cannot make any other moves…

To states (j)

From states (i)

$$\mathbf{A} = \begin{array}{cc} & \begin{matrix} 0 & \quad 1 & \quad 2 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \end{matrix} & \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix} \end{array}$$

Transition matrix for pump system



$$\mathbf{A} = \begin{array}{c} \\ \text{From states } (i) \end{array} \begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & -\lambda_A & \lambda_A & 0 \\ 1 & \mu_A & -(\mu_A + \lambda_B) & \lambda_B \\ 2 & \mu_{AB} & 0 & -\mu_{AB} \end{array}$$

To **states (j)**

Quantitative analysis based on Markov analysis

- Time dependent probabilities of being in a specific state, assuming what is the initial state (Kolmogorov forward equation):

$$\boxed{\mathbf{P(t)A} = d\mathbf{P(t)}/dt, \text{ where } \mathbf{P(t)}=[\mathrm{P}_0(\mathrm{t}), \mathrm{P}_1(\mathrm{t}),...,\mathrm{P}_\mathrm{r}(\mathrm{t})]}$$

Tells how the probability of being in specific states at t evolve over time.

- <u>Our focus:</u>
  - Average (steady state) probability of being in a specific state (as PFD and PFH are average measures):

$$\boxed{\mathbf{PA} = \mathbf{0}}$$

Simplification made for easier hand-calculation. We must always be in one of the available states:

$$\boxed{P_1 + P_2 + ... + \mathrm{P}_\mathrm{r} = 1 \rightarrow \mathbf{PA}^* = \mathbf{b}, \text{ with } \mathbf{b} = [0,0,...,1]}$$

A* is the **transition matrix where one column is replaced by 1's**

  - Frequency (v_j) of entering into a state j (time dependent or **average**):

$$\boxed{v_j = \sum_{i \neq j} P_i \times a_{ij}}$$

is equal to the sum of the probability of being in a state i and then jumping into state j

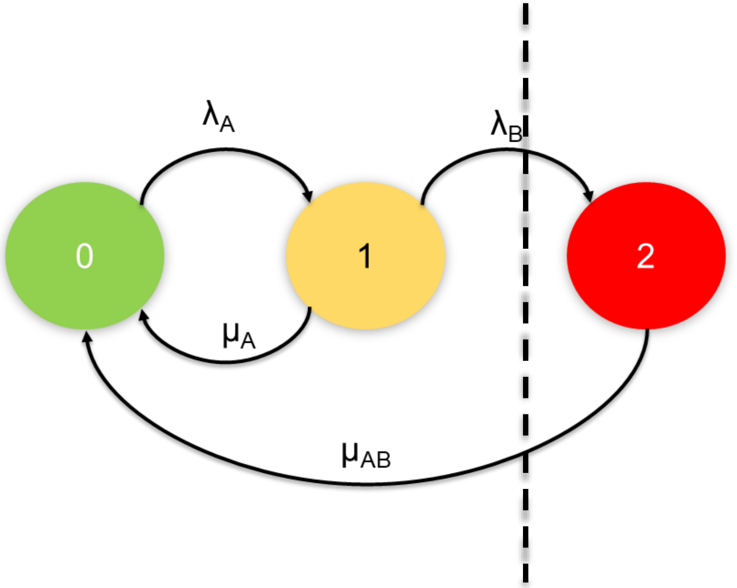
$$\mathbf{A}^* = \begin{bmatrix} a_{00} & a_{01} & ... & 1 \\ a_{10} & a_{11} & ... & 1 \\ a_{20} & a_{21} & ... & 1 \\ a_{30} & a_{31} & ... & 1 \end{bmatrix}$$

Example

Example: Steady state calculations

- Equation to solve becomes:

$$\begin{bmatrix} P_0 & P_1 & P_2 \end{bmatrix} \begin{bmatrix} -\lambda_A & \lambda_A & 1 \\ \mu_A & -(\mu_A + \lambda_B) & 1 \\ \mu_{AB} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$



- Steady state solution becomes (after a bit of work):

$$P_0 = \frac{\mu_{AB}(\lambda_B + \mu_A)}{(\lambda_B + \mu_{AB})\lambda_A + (\lambda_{\mathbf{B}} + \mu_A)\mu_{AB}}$$

$$P_1 = \frac{\mu_{AB}\lambda_A}{(\lambda_B + \mu_{AB})\lambda_A + (\lambda_B + \mu_A)\mu_{AB}}$$

Average probability of being in failed state

$$\boxed{P_2 = \frac{\lambda_A \lambda_B}{(\lambda_B + \mu_{AB})\lambda_A + (\lambda_B + \mu_A)\mu_{AB}}}$$

$$\boxed{\upsilon_2 = P_1 \cdot \lambda_B}$$

Frequency of entering the failed state

Puh… but with a tool we can consentrate on A not A*

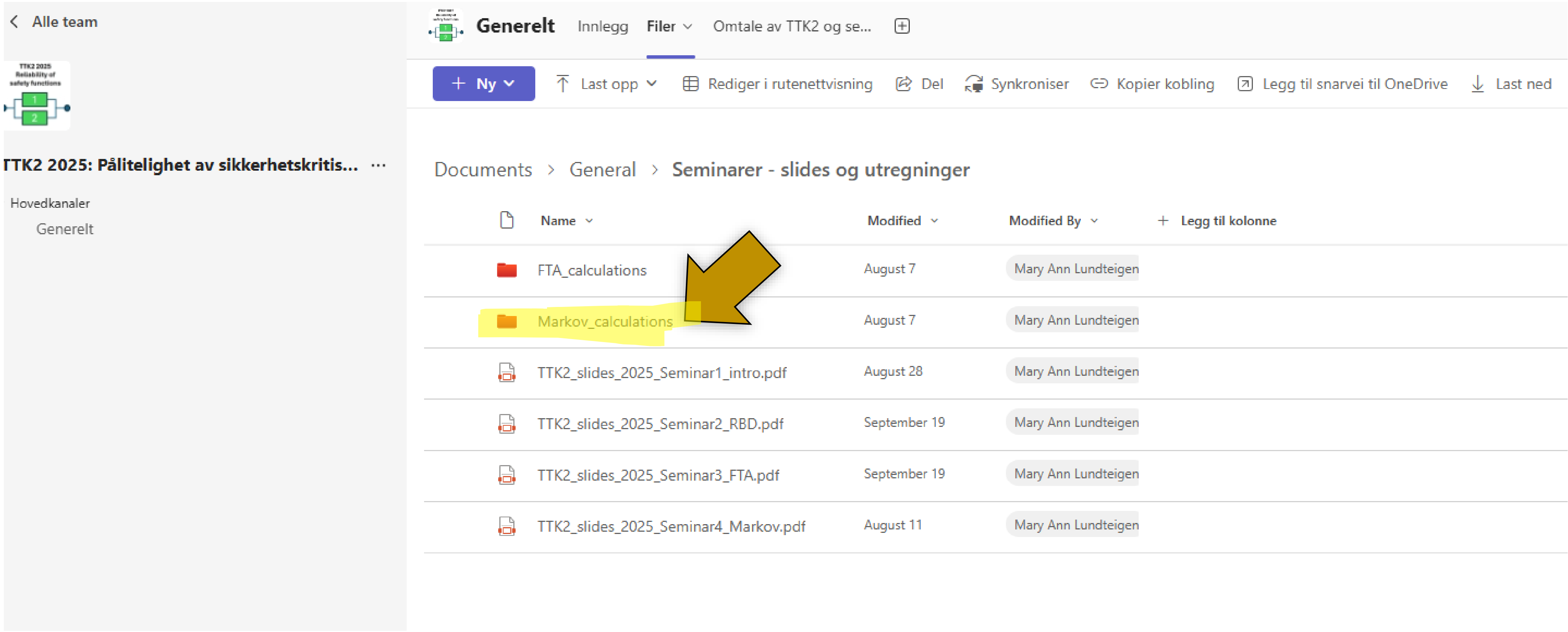
- We are using an excel based tool developed by Professor Jørn Vatn at department of Mechanical and Industrial Engineering.

  MarkovGeneralJVatn_2024_empty.xlsm

- Here, we will define transition rates, the dimension of the matrix, and the entries of the transition matrix A.

- We can also get the plot for how a specific state is reaching its steady state probability.

Alle utregninger med regneark finner dere her



Software tool: MarkovGeneralJVatn_2024_empty.xlsm

Må også velge en simulerings-tid stor nok til at steady state oppnås. Notasjon i samsvar med rater

Dimensjon er antall rader/kolonner i A-matrisen

Steady state probabilities calculted from A matrix directly

Fyller inn transisjoner i matrise A. <u>IKKE fyll inn i diagonal-elementene!</u> Det beregnes automatisk av programmet men vises ikke

Må si fra hva som er start-tilstand. Typisk når systemet virker.

Transisjons-rater

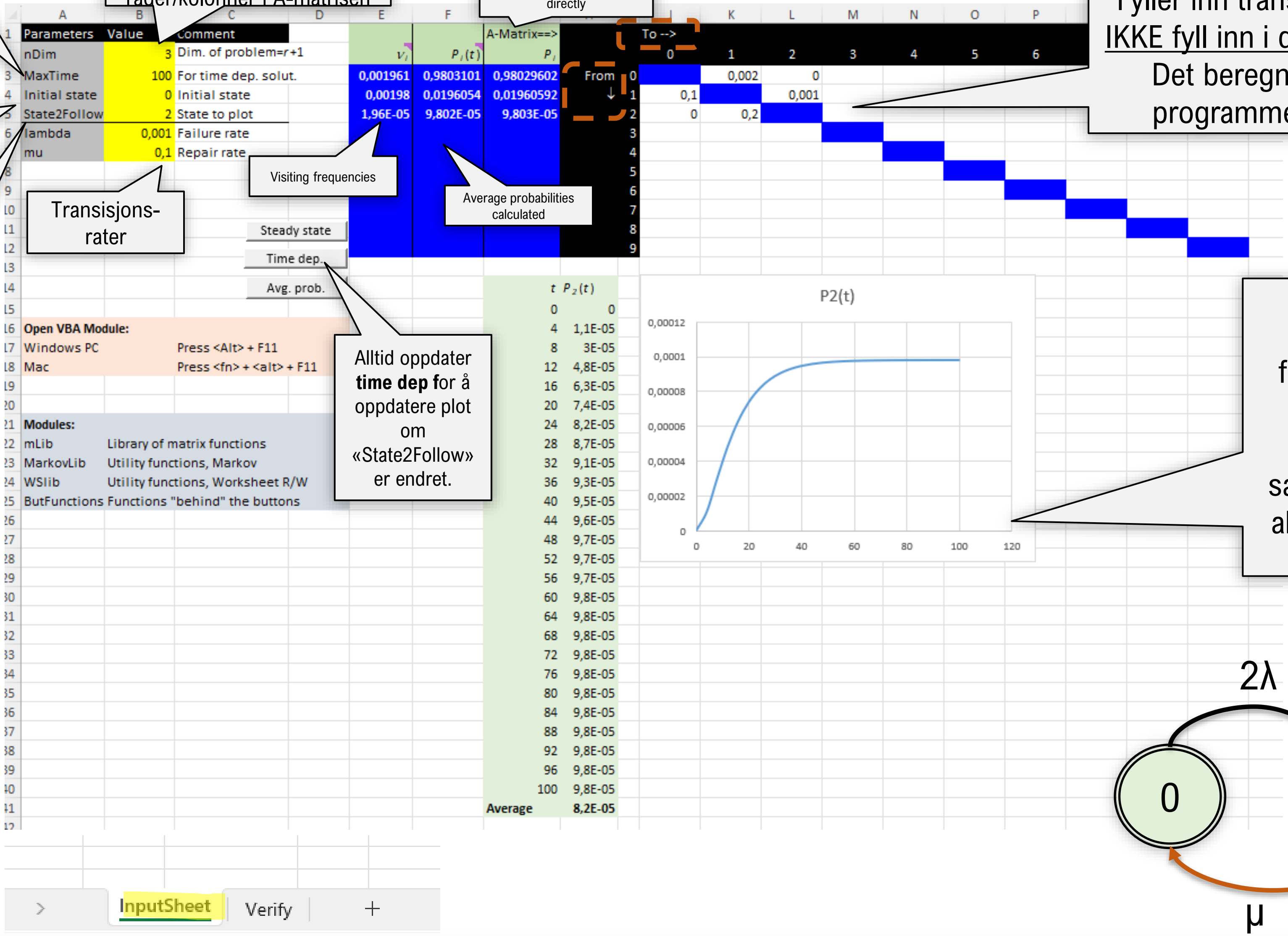
Visiting frequencies

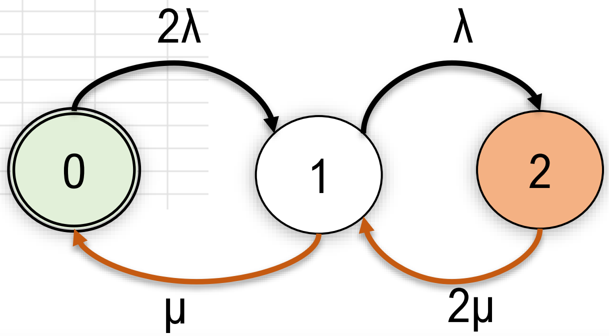
Average probabilities calculated

Må velge hvilken tilstand å følge for plottet. Kan endres på!

Alltid oppdater **time dep** for å oppdatere plot om «State2Follow» er endret.

Plotter tilstanden man følger. Ser hvordan man når steady state (gjennomsnittlig sannsynlighet for å være i akkurat denne tilstanden)





Software tool: MarkovGeneralJVatn_2024_empty.xlsm

Check if the transition matrix is correct

		To -->		
		0	1	2
From	0		=2*lambda	0
↓	1	=mu		=lambda
	2	0	=2*mu	
	3			
	4			
	5			
	6			
	7			
	8			
	9			

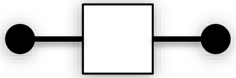
InputSheet | Verify | +

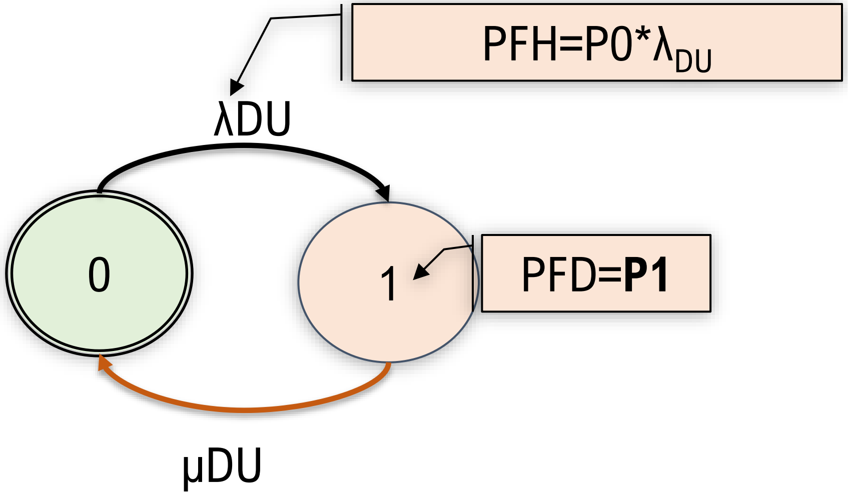
lambda | f_x | 0,001

	A	B	C
1	Parameters	Value	Comment
2	nDim	3	Dim. of problem=r+1
3	MaxTime	100	For time dep. solut.
4	Initial state	0	Initial state
5	State2Follow	2	State to plot
6	lambda	0,001	Failure rate
7	mu	0,1	Repair rate
8			

For at det skal stå tekst i cellene her, må cellen som tallverdien ble lagt inn i være navngitt. Celle B5 er eksempelvis gitt navnet «lambda».

What is PFD and PFH in Markov model?

Example:  Single subsystem (component)

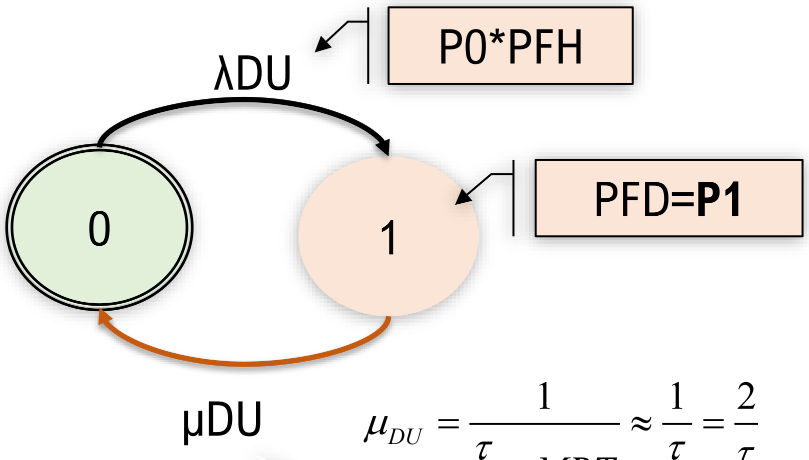


PFD: Sum of average (i.e., steady state) probability of staying failed states (P_i's).
PFH: Sum of average rates of jumping into a failed state (visiting frequencies, v_i's)

Markov analysis of SIF: 1oo1 with DU failures

State	Description
0	Component OK
1	Has a DU failure

$$\mathbf{A} = \begin{bmatrix} -\lambda_{DU} & \lambda_{DU} \\ \mu_{DU} & -\mu_{DU} \end{bmatrix}$$



$$\mu_{DU} = \frac{1}{\frac{\tau}{2} + MRT} \approx \frac{1}{\frac{\tau}{2}} = \frac{2}{\tau}$$

Parameter	Description	Value
λDU	DU failure rate	1E-6 per your
μDU	DU **restoration** rate. Inverse av “restoration time”. Equal to **2/τ** which is the mean time to restore **one** DU failure, where τ is the test interval	2/8760 hours
τ	Regular test interval	8760 hours

In fact, we are cheating a bit for DU restoration rates! τ/2 is **not** a **statistical mean** as τ is **deterministic** and not given by a distribution… But it has been shown that the cheating does not lead to much error. And pragmatically we estimate τ/2 as a mean downtime given that a DU failure occured

Restoration time dekker både tid til deteksjon samt reparasjonstid. Med hensyn på DU feil er det midlere nedetid før feilen detekteres (gitt av τ/2 for en komponent) + midlere reparasjonstid om det er avdekket en feil (MRT – mean time to repair): Fordi MRT ofte er noen få timer (kanskje opp til et døgn) mens τ kan være fra noen få måneder til et par år, så er τ/2 >>MRT

Markov analysis of SIF: 1oo1 with DU failures

PFH

PFD

nDim	2	Dim. of problem=r+1
MaxTime	35000	For time dep. solut.
Initial state	0	Initial state
State2Follow	1	State to plot
LDD	0,00E+00	Failure rate per hour
LDU	1,00E-06	Failure rate per hour
muDD	0	repair rate per hour
muDU	0,0002283	repair rate per hour
tau	8760	hours

Steady state

Time dep.

Avg. prob.

Open VBA Module:

Windows PC	Press <Alt> + F11
Mac	Press <fn> + <alt> + F11

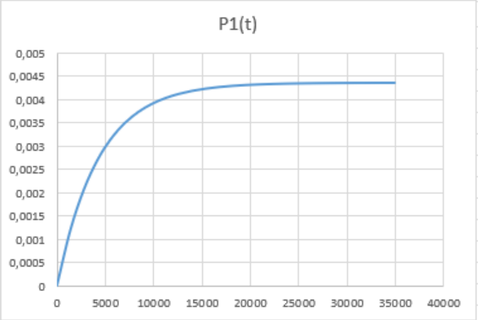
Modules:

mLib	Library of matrix functions
MarkovLib	Utility functions, Markov
WSlib	Utility functions, Worksheet R/W
ButFunctions	Functions "behind" the buttons

v_i	$P_i(t)$	P_i
9,96E-07	0,9956424	0,99563909
9,96E-07	0,0043595	0,0043609

		0	1	2	3	4	5	6	7	8	9
From	0		0,000001	0							
↓	1	0,000228		0							
	2		0								
	3		0								
	4										
	5										
	6										
	7										
	8										
	9										

t	$P_1(t)$
0	0
1400	0,0012
2800	0,00207
4200	0,0027
5600	0,00316
7000	0,00349
8400	0,00373
9800	0,0039
11200	0,00403
12600	0,00412
14000	0,00419
15400	0,00423
16800	0,00427
18200	0,00429
19600	0,00431
21000	0,00433
22400	0,00434
23800	0,00434
25200	0,00435
26600	0,00435
28000	0,00435
29400	0,00436
30800	0,00436
32200	0,00436
33600	0,00436
35000	0,00436
Average	0,00375



PFD: P1= 4.36E-3
PFH: v1=9.96E-7 per hour

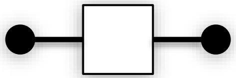
For comparison:

$$PFD \approx \frac{\lambda_{DU}\tau}{2} = \frac{1E-6 \cdot 8760}{2} = 4.38E-3$$

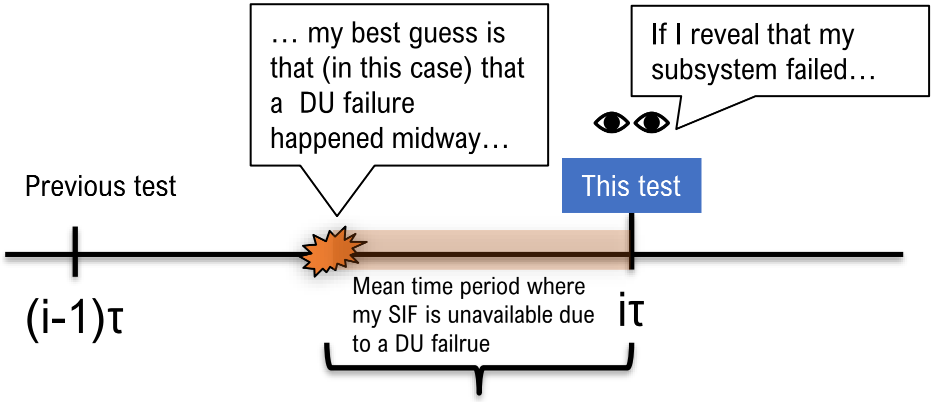
$$PFH \approx 1E-6 \text{ per hour}$$

About mean restoration time for DU failures

What are the **mean restoration times for DU failures** for subsystems with different voting?



Single subsystem (component)



$$T_{DU} = \frac{\tau}{2} + MRT \approx \frac{\tau}{2}$$

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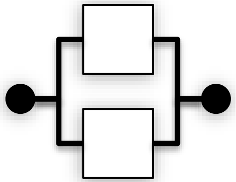
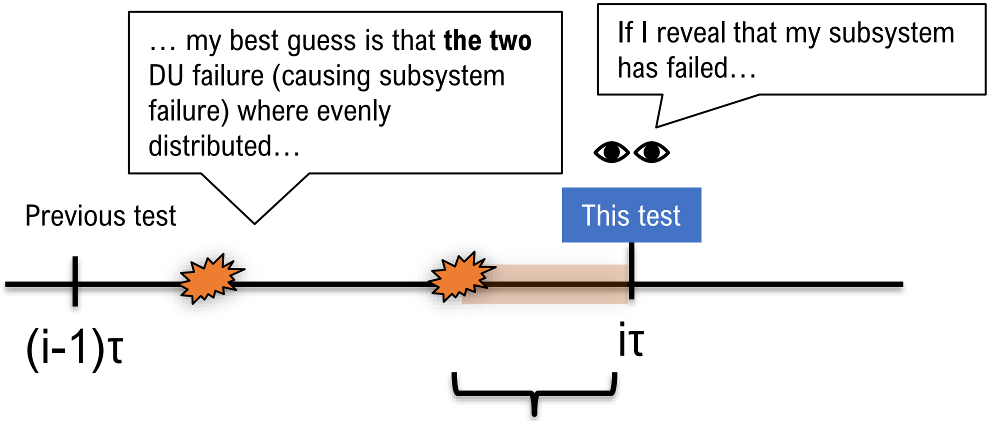
About mean restoration time for DU failures

What are the **mean restoration times T_{DU}** for subsystems with different voting?



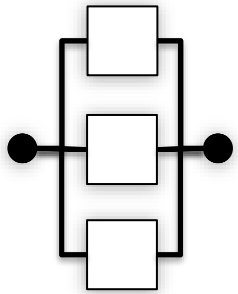
1oo2 voted subsystem



$$T_{DU} = \frac{\tau}{3} + MRT \approx \frac{\tau}{3}$$

About mean restoration time for DU failures

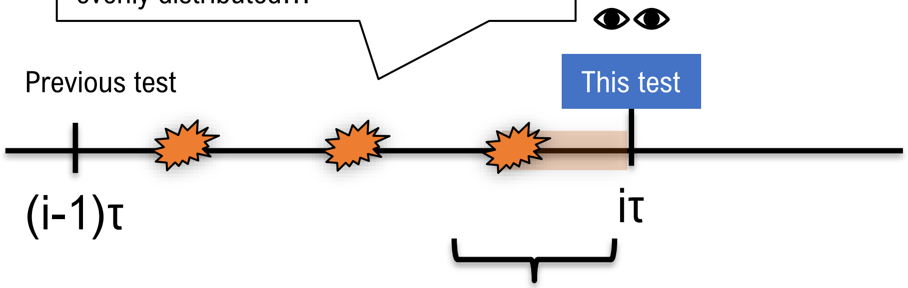
What are the **mean restoration times T_{DU}** for subsystems with different voting?



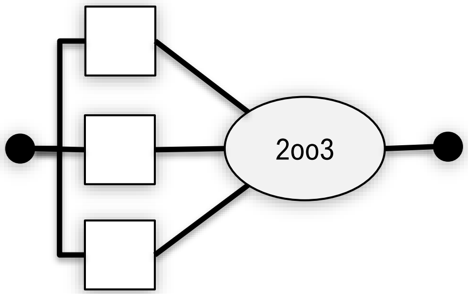
1oo3 voted subsystem

… my best guess is that **the three** DU failures (the minimum # of DU failures required for the subsystem to fail) where evenly distributed…

If I reveal that my subsystem has failed…



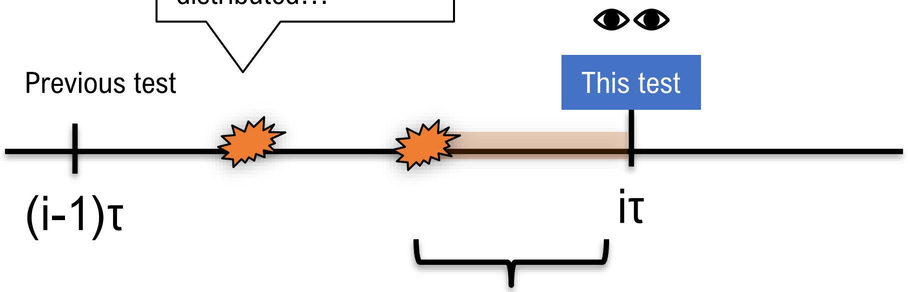
$$T_{DU} = \frac{\tau}{4} + MRT \approx \frac{\tau}{4}$$



2oo3 voted subsystem

… my best guess is that **the two** DU failure where evenly distributed…

If I reveal that my subsystem has failed…

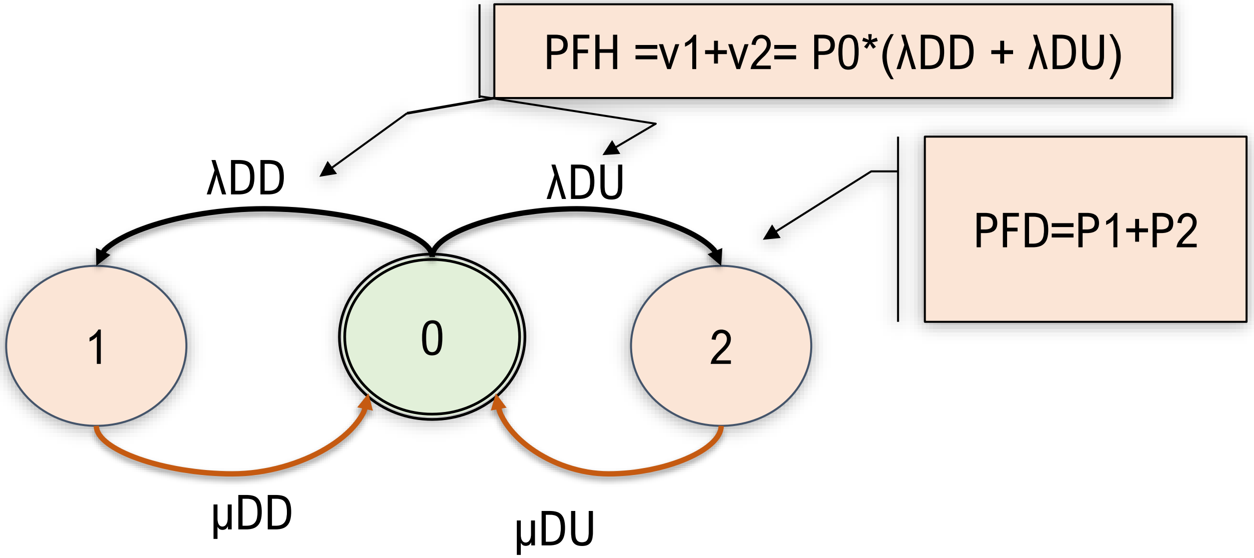


$$T_{DU} = \frac{\tau}{3} + MRT \approx \frac{\tau}{3}$$

Markov analysis of SIF: 1oo1 with DD and DU failures

State	Description
0	Component OK
1	Has a DD failure
2	Has a DU failure

Parameter	Description	Value
λDD	DD failure rate	3E-6 per hour
λDU	DU failure rate	1E-6 per your
μDD	1/MTTR, where MTTR is mean time to restoration of one DD failure (MTTR a term used by IEC 61508). Detection time is often in seconds, repair time similar to MRT for DU failures, i.e., usually a few hours.	1/24 hours
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval	2/8760 hours
τ	Regular test interval	8760 hours



$$\mathbf{A} = \begin{bmatrix} -(\lambda_{DD} + \lambda_{DU}) & \lambda_{DD} & \lambda_{DU} \\ \mu_{DD} & -\mu_{DD} & 0 \\ \mu_{DU} & 0 & -\mu_{DU} \end{bmatrix}$$

Markov analysis of SIF: 1oo1 with DD and DU failures

Parameters	Value	Comment
nDim	3	Dim. of problem=r+1
MaxTime	35040	For time dep. solut.
Initial state	0	Initial state
State2Follow	2	State to plot
LDD	3,00E-06	Failure rate per hour
LDU	1,00E-06	Failure rate per hour
muDD	0,0833333	repair rate per hour
muDU	0,0002283	repair rate per hour
tau	8760	hours

Steady state

Time dep.

Avg. prob.

v_i	$P_i(t)$	P_i
3,98E-06	0,9961304	0,99560344
2,99E-06	3,566E-05	3,5842E-05
9,96E-07	0,0038192	0,00436074

A-Matrix==>

From ↓ / To -->	0	1	2	3	4	5	6	7	8	9
0		0,000003	0,000001	0						
1	0,083333		0	0						
2	0,000228	0								
3		0								
4										
5										
6										
7										
8										
9										

t	$P_2(t)$
0	0
1401,59998	0,0012
2803,19971	0,00207
4204,79932	0,0027
5606,39893	0,00316
7007,99854	0,00349
8409,59863	0,00373
9811,2002	0,0039
11212,8018	0,00403
12614,4033	0,00412
14016,0049	0,00419
15417,6064	0,00423
16819,207	0,00427
18220,8086	0,00429
19622,4102	0,00431
30835,2227	0,00436
32236,8242	0,00436
33638,4219	0,00436
35040,0156	0,00436
Average	**0,00375**

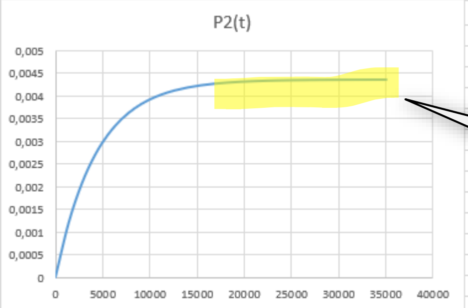
Open VBA Module:

Windows PC — Press <Alt> + F11

Mac — Press <fn> + <alt> + F11

Modules:

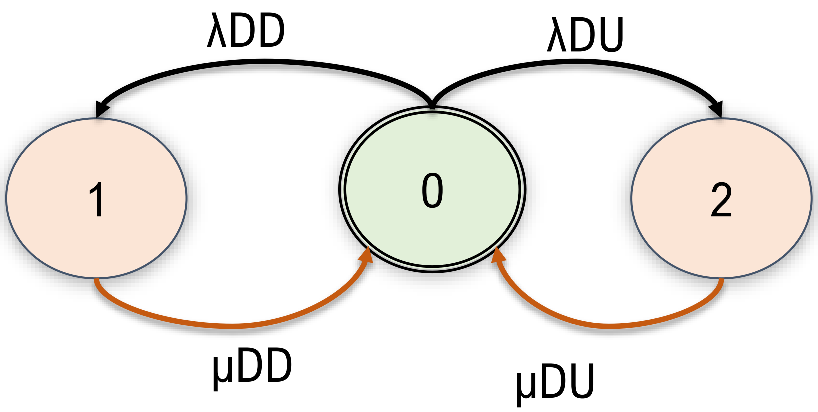
mLib	Library of matrix functions
MarkovLib	Utility functions, Markov
WSlib	Utility functions, Worksheet R/W
ButFunctions	Functions "behind" the buttons



Får bare plottet en tilstand om gangen

PFD: P1+P2 = 3.58E-5+4.36E-3=4.40E-3

PFH: v1+v2=P0*(λDD+λDU)=3.98E-6 per hour



Summering gjøres manuelt

For comparison (simplified formulas, seminar 2):

$$PFD \approx \frac{\lambda_{DU}\tau}{2} = \frac{1E-6 \cdot 8760}{2} = 4.38E-3$$

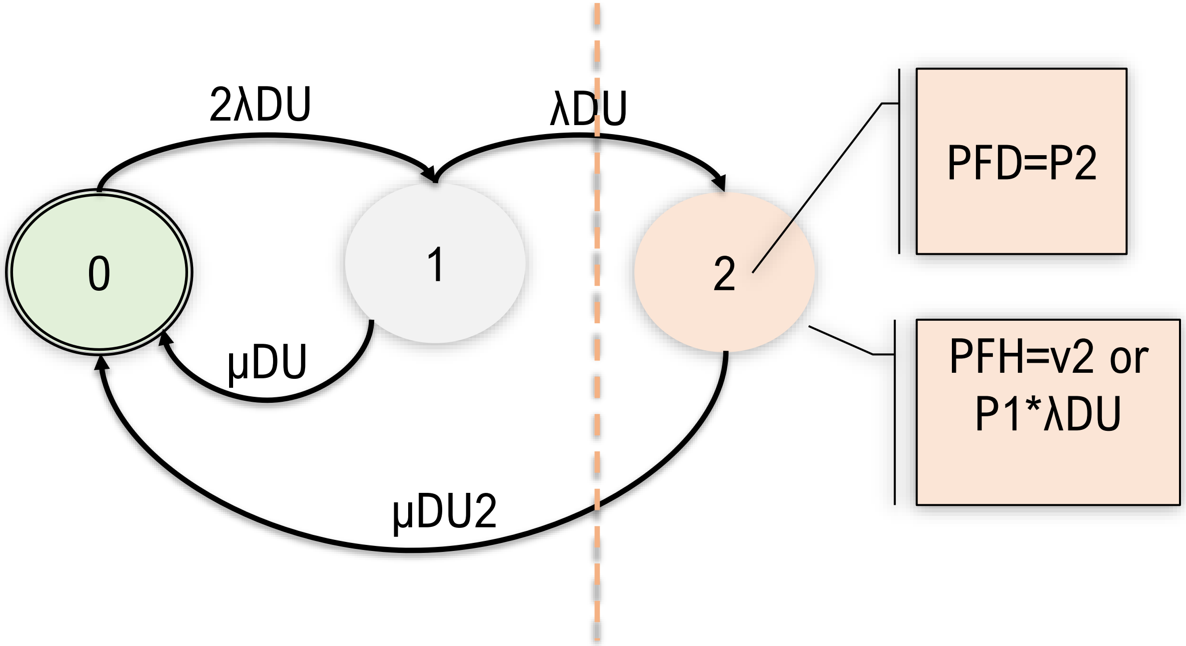
$$PFH = \lambda_{DU} + \lambda_{DD} = 4E-6 \text{ per hour}$$

Markov analysis of SIF: 1oo2 with DU only

State	Description
0	Both components OK
1	1 DU, 1 OK
2	2 DU

Parameter	Description	Value
λDU	DU failure rate	1E-6 per hour
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval	2/8760 hours
μDU2	3/τ which is the mean time to restore TWO DU failures, where τ is the test interval	3/8760 hours
tau	Test interval	8760 hours

Forklares i forelesningen!



$$\mathbf{A} = \begin{bmatrix} -2\lambda_{DU} & 2\lambda_{DU} & 0 \\ \mu_{DU} & -(\mu_{DU} + \lambda_{DU}) & \lambda_{DU} \\ \mu_{DU2} & 0 & -\mu_{DU2} \end{bmatrix}$$

Markov analysis of SIF: 1oo2 with DU only

Parameters	Value	Comment
nDim	3	Dim. of problem=r+1
MaxTime	35040	For time dep. solut.
Initial state	0	Initial state
State2Follow	2	State to plot
LDD	0,00E+00	Failure rate per hour
LDU	1,00E-06	Failure rate per hour
muDD	0	repair rate per hour
muDU	0,0002283	repair rate per hour
muDU2	0,0003425	repair rate per hour
tau	8760	hours

Steady state

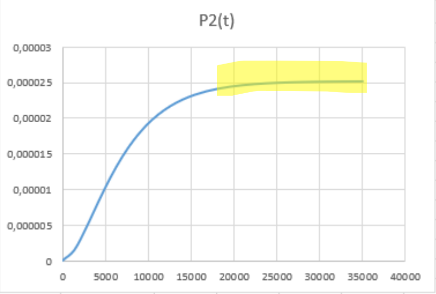
Time dep.

Avg. prob.

v_i	$P_i(t)$	A-Matrix==> P_i
1,98E-06	0,9923892	0,99132854
1,98E-06	0,0075819	0,00864617
8,65E-09	2,004E-05	2,5247E-05

	To --> 0	1	2	3	4	5	6	7	8	9
From ↓ 0		0,000002	0							
1	0,000228		0,000001							
2	0,000342	0								
3										
4										
5										
6										
7										
8										
9										

t	$P_2(t)$
0	0
1401,59998	1,5E-06
2803,19971	4,7E-06
4204,79932	8,3E-06
5606,39893	1,2E-05
7007,99854	1,5E-05
8409,59863	1,7E-05
9811,2002	1,9E-05
11212,8018	2,1E-05
12614,4033	2,2E-05
14016,0049	2,3E-05
15417,6064	2,3E-05
16819,207	2,4E-05
18220,8086	2,4E-05
19622,4102	2,4E-05
21024,0117	2,5E-05
22425,6133	2,5E-05
23827,2148	2,5E-05
25228,8164	2,5E-05
26630,418	2,5E-05
28032,0195	2,5E-05
29433,6211	2,5E-05
30835,2227	2,5E-05
32236,8242	2,5E-05
33638,4219	2,5E-05
35040,0156	2,5E-05
Average	2E-05



Open VBA Module:

Windows PC	Press <Alt> + F11
Mac	Press <fn> + <alt> + F11

Modules:

mLib	Library of matrix functions
MarkovLib	Utility functions, Markov
WSlib	Utility functions, Worksheet R/W
ButFunctions	Functions "behind" the buttons

PFD=P2≈2.52E-5
PFH= 8.65E-9 per hour

For comparison:

$$PFD \approx \frac{(\lambda_{DU}\tau)^2}{3} = \frac{(1E-6\cdot 8760)^2}{3} \approx 2.56E-5$$

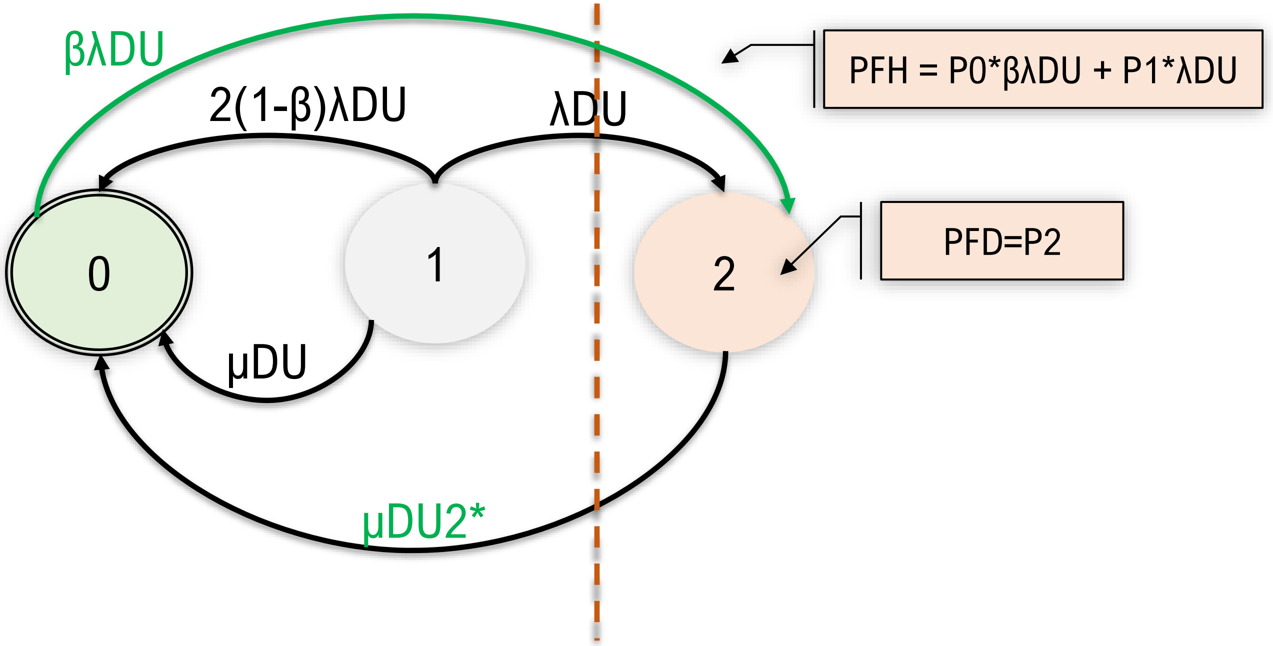
$$PFH \approx \lambda_{DU}^2\tau = 8.76E-9 \text{ per hour}$$

Markov analysis of SIF: 1oo2 with DU only + CCF

State	Description
0	Component OK
1	1 DU, 1 OK
2	2 DU

Parameter	Description	Value
λDU	DU failure rate	1E-6 per hour
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval	2/8760 hours
μDU2*	2/τ which is the mean time to restore a CCF failure	2/8760 hours
tau	Test interval	8760 hours

Forklares i forelesningen!



$$\mathbf{A} = \begin{bmatrix} -2(1-\beta)\lambda_{DU} & 2(1-\beta)\lambda_{DU} & \beta\lambda_{DU} \\ \mu_{DU} & -(\mu_{DU}+\lambda_{DU}) & \lambda_{DU} \\ \mu_{DU2*} & 0 & -\mu_{DU2*} \end{bmatrix}$$

Markov analysis of SIF: 1oo2 with DU only + CCF

Parameters	Value	Comment
nDim	3	Dim. of problem=r+1
MaxTime	35040	For time dep. solut.
Initial state	0	Initial state
State2Follow	2	State to plot
LDD	0,00E+00	Failure rate per hour
LDU	1,00E-06	Failure rate per hour
muDD	0	repair rate per hour
muDU	0,0002283	repair rate per hour
muDU2	0,0002283	repair rate per hour
tau	8760	hours
beta	5 %	CCF fraction

Steady state

Time dep.

Avg. prob.

v_i	$P_i(t)$	P_i
1,93E-06	9,93E-01	9,92E-01
1,88E-06	7,20E-03	8,22E-03
5,78E-08	2,17E-04	2,53E-04

A-Matrix==>

From ↓ / To -->	0	1	2	3	4	5	6	7	8	9
0		0,0000019	0,00000005							
1	0,000228		0,000001							
2	0,000228	0								
3										
4										
5										
6										
7										
8										
9										

Open VBA Module:

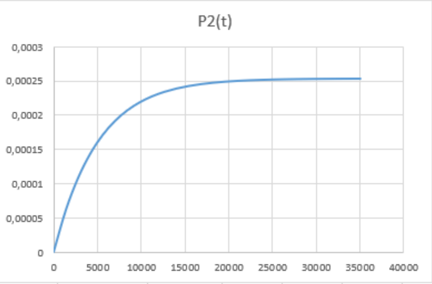
Windows PC — Press <Alt> + F11

Mac — Press <fn> + <alt> + F11

Modules:

mLib	Library of matrix functions
MarkovLib	Utility functions, Markov
WSlib	Utility functions, Worksheet R/W
ButFunctions	Functions "behind" the buttons

t	$P_2(t)$
0	0
1401,59998	6,2E-05
2803,19971	1,08E-04
4204,79932	1,44E-04
5606,39893	1,71E-04
7007,99854	1,91E-04
8409,59863	2,07E-04
9811,2002	2,18E-04
11212,8018	2,27E-04
12614,4033	2,34E-04
14016,0049	2,39E-04
15417,6064	2,42E-04
16819,207	2,45E-04
18220,8086	2,47E-04
19622,4102	2,49E-04
21024,0117	2,50E-04
22425,6133	2,51E-04
23827,2148	2,51E-04
25228,8164	2,52E-04
26630,418	2,52E-04
28032,0195	2,52E-04
29433,6211	2,53E-04
30835,2227	2,53E-04
32236,8242	2,53E-04
33638,4219	2,53E-04
35040,0156	2,53E-04
Average	2,14E-04



PFD=P2≈2.53E-4

PFH=v2=5.78E-8 per hour

For comparison:

$$PFD \approx \frac{((1-\beta)\lambda_{DU}\tau)^2}{3} + \frac{\beta\lambda_{DU}\tau}{2} \approx 2.42E-4$$

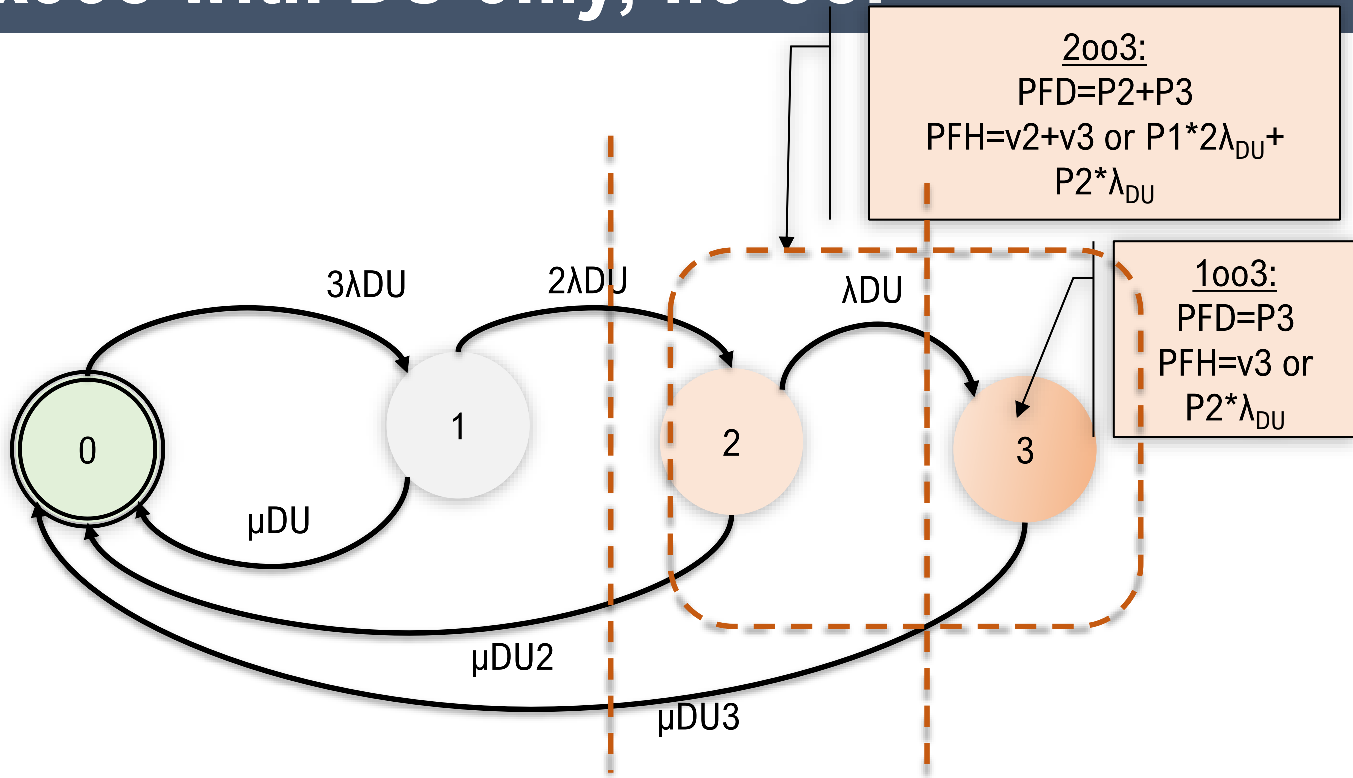
$$PFH \approx ((1-\beta)\lambda_{DU})^2\tau + \beta\lambda_{DU} \approx 5.79E-8 \text{ per hour}$$

Markov analysis with xoo3 with DU only, no CCF

State	Description
0	Component OK
1	1 DU, 2 OK
2	2 DU, 1OK
3	3DU

Parameter	Description	Value
λDU	DU failure rate	1E-6 per hour
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval	2/8760 hours
μDU2	3/τ which is the mean time to restore TWO DU failures, where τ is the test interval	3/8760 hours
μDU3	4/τ which is the mean time to restore THREE DU failures, where τ is the test interval	4/8760 hours
τ	Regular test interval	8760 hours

Forklares i forelesningen!



$$\mathbf{A} = \begin{bmatrix} -3\lambda_{DU} & 3\lambda_{DU} & 0 & 0 \\ \mu_{DU} & -(\mu_{DU} + 2\lambda_{DU}) & 2\lambda_{DU} & 0 \\ \mu_{DU2} & 0 & -(\mu_{DU2} + \lambda_{DU}) & \lambda_{DU} \\ \mu_{DU3} & 0 & 0 & -\mu_{DU3} \end{bmatrix}$$

Markov analysis with xoo3 with DU only, no CCF

Parameters	Value	Comment
nDim	4	Dim. of problem=r+1
MaxTime	43800	For time dep. solut.
Initial state	0	Initial state
State2Follow	3	State to plot
LDD	0,00E+00	Failure rate per hour
LDU	1,00E-06	Failure rate per hour
muDD	0	repair rate per hour
muDU	0,0002283	repair rate per hour
muDU2	0,0003425	repair rate per hour
muDU3	0,0004566	repair rate per hour
tau	8760	hours

Steady state

Time dep.

Avg. prob.

v_i	$P_i(t)$	P_i
2,96E-06	0,9883271	0,98706752
2,96E-06	0,0116023	0,01285744
2,57E-08	6,259E-05	7,4869E-05
7,49E-11	1,289E-07	1,6405E-07

A-Matrix==>

From ↓ / To -->	0	1	2	3	4	5	6	7	8	9
0		0,000003	0	0						
1	0,000228		0,000002	0						
2	0,000342	0		0,000001						
3	0,000457	0	0							
4										
5										
6										
7										
8										
9										

t	$P_3(t)$
0	0
1752	3,4E-09
3504	1,8E-08
5256	4E-08
7008	6,5E-08
8760	8,8E-08
10512	1,1E-07
12264	1,2E-07
14016	1,3E-07
15768	1,4E-07
17520	1,5E-07
19272	1,5E-07
21024	1,6E-07
22776	1,6E-07
24528	1,6E-07
26280	1,6E-07
28032	1,6E-07
29784	1,6E-07
31536	1,6E-07
33288	1,6E-07
35040	1,6E-07
36792	1,6E-07
38544	1,6E-07
40296	1,6E-07
42048	1,6E-07
43800	1,6E-07
Average	1,3E-07

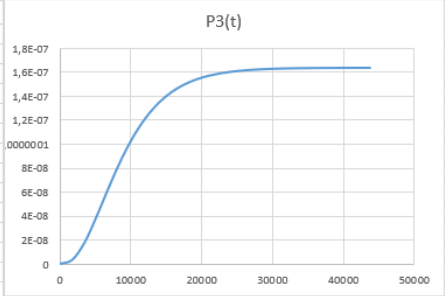
Open VBA Module:

Windows PC — Press <Alt> + F11

Mac — Press <fn> + <alt> + F11

Modules:

mLib	Library of matrix functions
MarkovLib	Utility functions, Markov
WSlib	Utility functions, Worksheet R/W
ButFunctions	Functions "behind" the buttons



1oo3:
PFD= P3 ≈ 1.64E-7
PFH=v3=7.49E-11 per hour
2oo3:
PFD=P2 + P3 ≈ 7.50E-5
PFH=v3+v4 ≈ 2.58E-8 per hour

For comparison:

1oo3:

$$PFD \approx \frac{(\lambda_{DU}\tau)^3}{4} = \frac{(1E-6 \cdot 8760)^3}{4} \approx 1.68E-7$$

$$PFH \approx \lambda_{DU}^3 \tau^2 \approx 7.49E-11 \text{ per hour}$$

2oo3:

$$PFD \approx (\lambda_{DU}\tau)^2 = (1E-6 \cdot 8760)^2 \approx 7.67E-5$$

$$PFH \approx 3\lambda_{DU}^2 \tau \approx 2.63E-8 \text{ per hour}$$

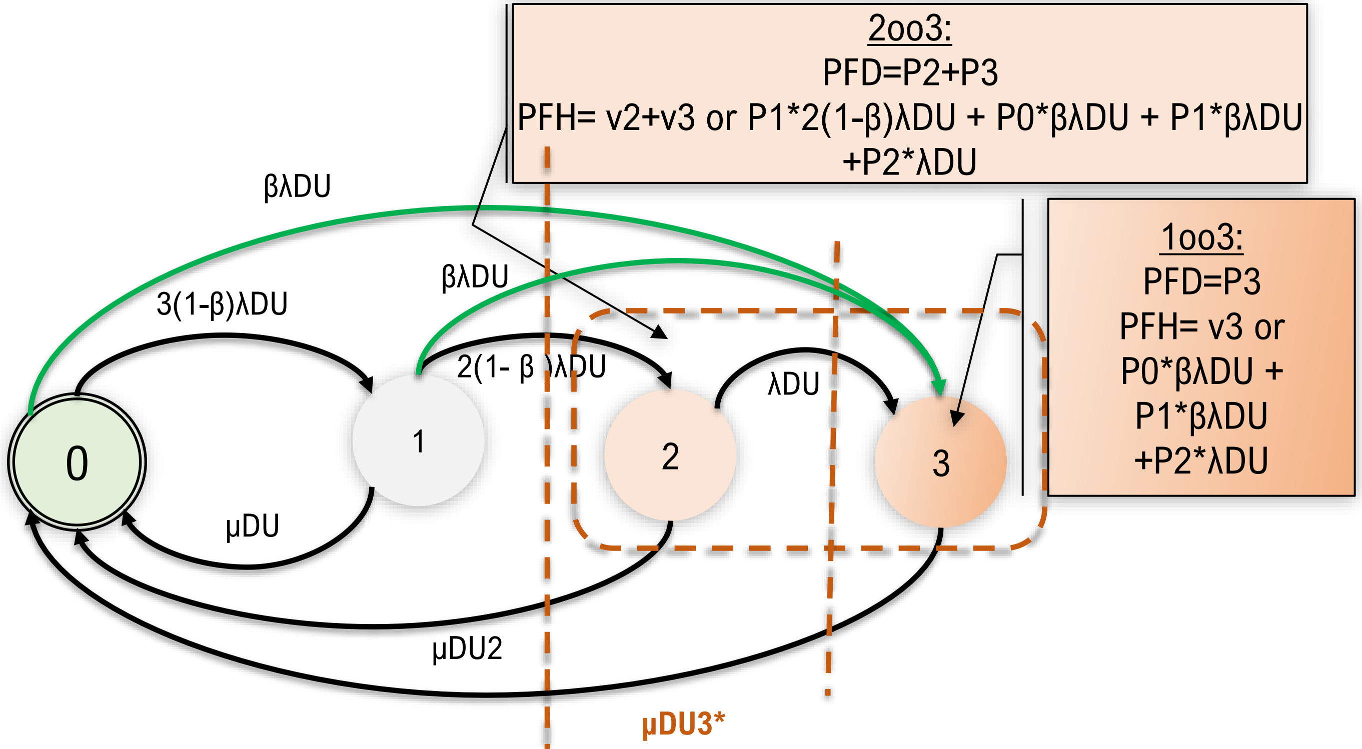
Ganske likt resultat. Markov gir litt lavere verdier da modellen hensyntar reparasjon ved 1 og 2 DU feil, ikke bare ved 3 DU feil.

Markov analysis with xoo3 with DU only, with CCF

State	Description
0	Component OK
1	1 DU, 2 OK
2	2 DU, 1OK
3	3DU

Parameter	Description	Value
λDU	DU failure rate	1E-6 per hour
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval	2/8760 hours
μDU2	3/τ which is the mean time to restore TWO DU failures, where τ is the test interval	3/8760 hours
μDU3*	**2/τ** which is the mean time to restore **a CCF, the dominating contributor** to downtime compared to independent failures.	**2**/8760 hours
tau	Test interval	8760 hours
β	Fraction of failure rate that is ommon cause failure (CCF)	5%

Forklares i forelesningen!



$$\mathbf{A} = \begin{bmatrix} -(3(1-\beta)\lambda_{DU} + \beta\lambda_{DU}) & 3(1-\beta)\lambda_{DU} & 0 & \beta\lambda_{DU} \\ \mu_{DU} & -(\mu_{DU} + 2(1-\beta)\lambda_{DU}) & 2(1-\beta)\lambda_{DU} & \beta\lambda_{DU} \\ \mu_{DU2} & 0 & -(\mu_{DU2} + \lambda_{DU}) & \lambda_{DU} \\ \mu_{DU3}* & 0 & 0 & -\mu_{DU3*} \end{bmatrix}$$

Markov analysis with xoo3 with DU only, with CCF

Parameters	Value	Comment
nDim	4	Dim. of problem=r+1
MaxTime	43800	For time dep. solut.
Initial state	0	Initial state
State2Follow	3	State to plot
LDD	0,00E+00	Failure rate per hour
LDU	1,00E-06	Failure rate per hour
muDD	0	repair rate per hour
muDU	0,0002283	repair rate per hour
muDU2	0,0003425	repair rate per hour
muDU3	0,0002283	repair rate per hour
beta	5 %	CCF fraction
tau	8760	hours

Steady state

Time dep.

Avg. prob.

Open VBA Module:

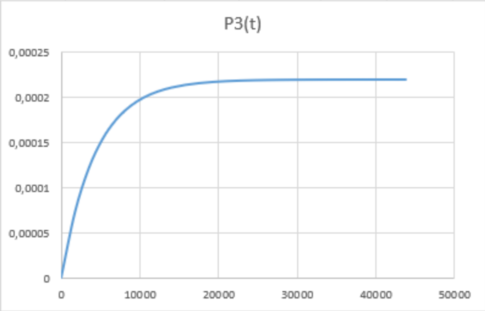
Windows PC	Press <Alt> + F11
Mac	Press <fn> + <alt> + F11

Modules:

mLib	Library of matrix functions
MarkovLib	Utility functions, Markov
WSlib	Utility functions, Worksheet R/W
ButFunctions	Functions "behind" the buttons

v_i	$P_i(t)$	A-Matrix==> P_i	From ↓	To --> 0	1	2	3	4	5	6	7	8	9
2,86E-06	9,89E-01	9,87E-01	0		0,00000285	0	0,00000005						
2,81E-06	1,10E-02	1,22E-02	1	0,000228		0,0000019	0,00000005						
2,32E-08	5,65E-05	6,76E-05	2	0,000342	0		0,000001						
5,01E-08	1,97E-04	2,19E-04	3	0,000228	0	0							
			4										
			5										
			6										
			7										
			8										
			9										

t	$P_3(t)$
0	0
1752	7,24E-05
3504	1,21E-04
5256	1,53E-04
7008	1,75E-04
8760	1,90E-04
10512	1,99E-04
12264	2,06E-04
14016	2,10E-04
15768	2,13E-04
17520	2,15E-04
19272	2,17E-04
21024	2,17E-04
22776	2,18E-04
24528	2,18E-04
26280	2,19E-04
28032	2,19E-04
29784	2,19E-04
31536	2,19E-04
33288	2,19E-04
35040	2,19E-04
36792	2,19E-04
38544	2,19E-04
40296	2,19E-04
42048	2,19E-04
43800	2,19E-04
Average	1,94E-04



1oo3:

PFD= P3 ≈ 2.19E-4

PFH=v3 ≈ 5.01E-8 per hour

2oo3:

PFD=P2 + P3 ≈ 2.87E-4

PFH=v1+v2 ≈7.33E-6 per hour

For comparison:

1oo3:

$$PFD \approx \frac{((1-\beta)\lambda_{DU}\tau)^3}{4} + \frac{\beta\lambda_{DU}\tau}{2} \approx 2.19E-4$$

$$PFH \approx \left((1-\beta)\lambda_{DU}\right)^3 \tau^2 + \beta\lambda_{DU} \approx 5.01E-8 \text{ per hour}$$

2oo3:

$$PFD \approx \left((1-\beta)\lambda_{DU}\tau\right)^2 + \frac{\beta\lambda_{DU}\tau}{2} \approx 2.88E-4$$

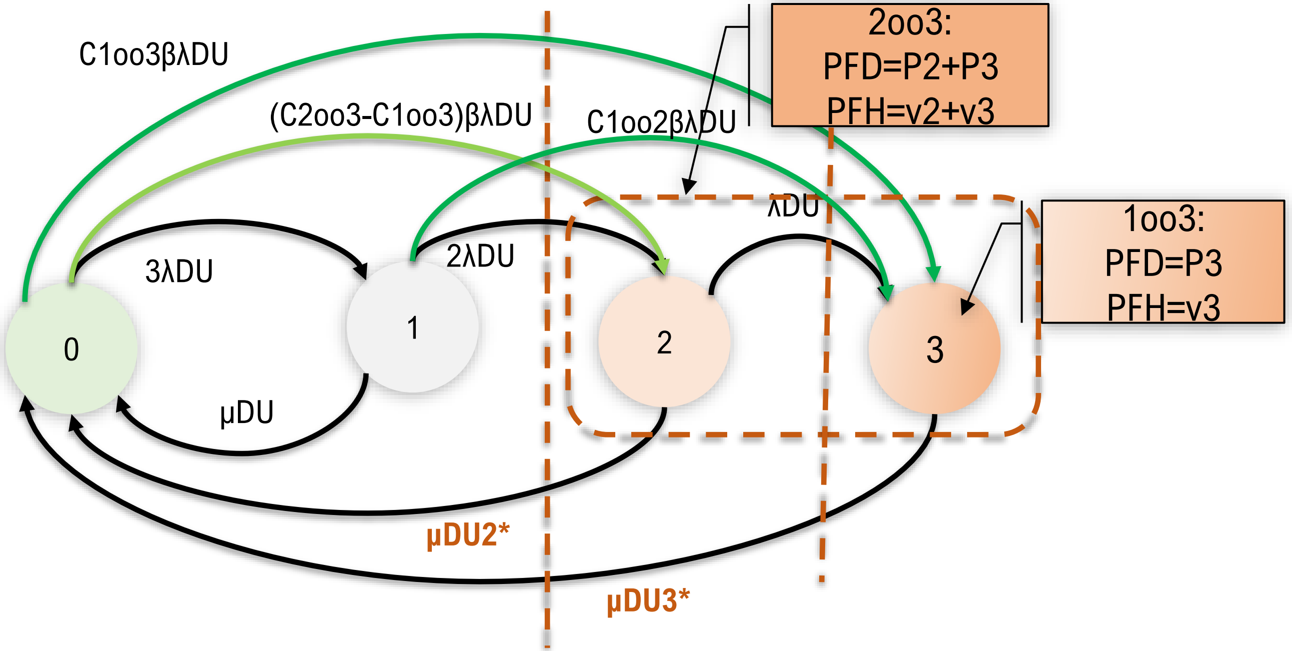
$$PFH \approx 3\left((1-\beta)\lambda_{DU}\right)^2 \tau + \beta\lambda_{DU} \approx 7.37E-8 \text{ per hour}$$

Liten forskjell i svarene også her.

Markov analysis with xoo3 with DU only, with CCF (PDS)

State	Description
0	Component OK
1	1 DU, 2 OK
2	2 DU, 1OK
3	3DU

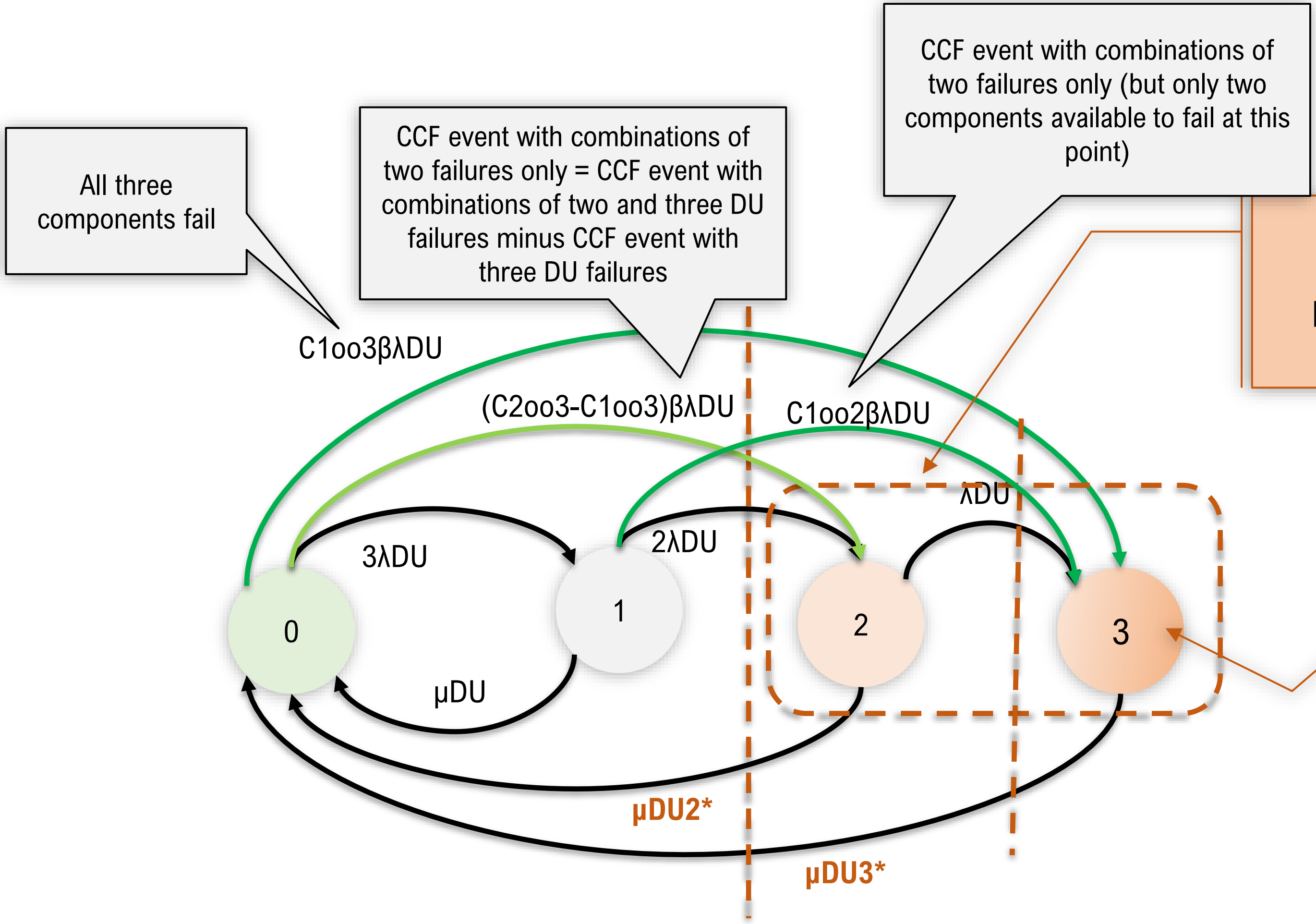
Paramter	Description
λDU	DU failure rate
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval
μDU2*	2/τ which is the mean time to restore **a CCF, the dominating contributor** to downtime compared to independent failures
μDU3*	2/τ which is the mean time to restore **a CCF, the dominating contributor** to downtime compared to independent failures



Forklares i forelesningen!

$$\mathbf{A} = \begin{bmatrix} -(3\lambda_{DU} + (C_{2oo3} - C_{1oo3})\beta\lambda_{DU} + C_{1oo3}\beta\lambda_{DU}) & 3\lambda_{DU} & (C_{2oo3} - C_{1oo3})\beta\lambda_{DU} & C_{1oo3}\beta\lambda_{DU} \\ \mu_{DU} & -(\mu_{DU} + 2\lambda_{DU}) & 2\lambda_{DU} & C_{1oo2}\beta\lambda_{DU} \\ \mu_{DU2*} & 0 & -(\mu_{DU2*} + \lambda_{DU}) & \lambda_{DU} \\ \mu_{DU3*} & 0 & 0 & -\mu_{DU3*} \end{bmatrix}$$

Explanations



<u>2oo3:</u>
PFD=P2+P3
PFH= v2+v3 <u>or</u> $P1*2\lambda_{DU}+P0*(C2oo3-C1oo2)\beta\lambda_{DU} + P0*C1oo3\ \beta\lambda_{DU} + P1*C1oo2\ \beta\lambda_{DU} + P2*\lambda_{DU}$)

<u>1oo3:</u>
PFD=P3
PFH= v3 or $P0*C1oo3\ \beta\lambda_{DU} + P1*C1oo2\ \beta\lambda_{DU} + P2*\lambda_{DU}$

Markov analysis with xoo3 with DU only, with CCF (PDS)

Parameters	Value	Comment
nDim	4	Dim. of problem=r+1
MaxTime	43800	For time dep. solut.
Initial state	0	Initial state
State2Follow	3	State to plot
LDD	0,00E+00	Failure rate per hour
LDU	1,00E-06	Failure rate per hour
muDD	0	repair rate per hour
muDU	0,0002283	repair rate per hour
muDU2	0,0002283	repair rate per hour
muDU3	0,0002283	repair rate per hour
beta	5 %	CCF fraction
tau	8760	hours

Steady state

Time dep.

Avg. prob.

Open VBA Module:

Windows PC	Press <Alt> + F11
Mac	Press <fn> + <alt> + F11

Modules:

mLib	Library of matrix functions
MarkovLib	Utility functions, Markov
WSlib	Utility functions, Worksheet R/W
ButFunctions	Functions "behind" the buttons

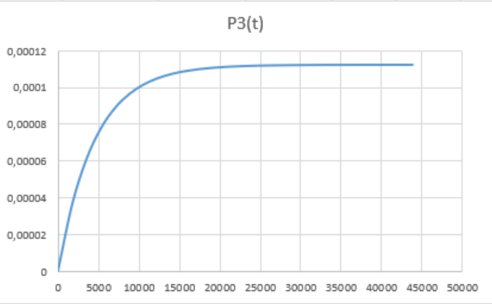
C1oo2	1
C1oo3	0,5
C2oo3	2

v_i	$P_i(t)$	P_i
3,06E-06	9,88E-01	9,87E-01
2,96E-06	1,16E-02	1,28E-02
9,97E-08	3,81E-04	4,35E-04
2,57E-08	1,01E-04	1,13E-04

A-Matrix==>

From ↓ / To -->	0	1	2	3	4	5	6	7	8	9
0		0,000003	0,000000075	0,000000025						
1	0,000228		0,000002	0,00000005						
2	0,000228	0		0,000001						
3	0,000228	0	0							
4										
5										
6										
7										
8										
9										

t	$P_3(t)$
0	0
1752	3,64E-05
3504	6,10E-05
5256	7,76E-05
7008	8,89E-05
8760	9,66E-05
10512	1,02E-04
12264	1,05E-04
14016	1,08E-04
15768	1,09E-04
17520	1,10E-04
19272	1,11E-04
21024	1,12E-04
22776	1,12E-04
24528	1,12E-04
26280	1,12E-04
28032	1,13E-04
29784	1,13E-04
31536	1,13E-04
33288	1,13E-04
35040	1,13E-04
36792	1,13E-04
38544	1,13E-04
40296	1,13E-04
42048	1,13E-04
43800	1,13E-04
Average	**9,93E-05**



1oo3:
PFD= P3 ≈ 1.13E-4
PFH=v3 ≈2.57E-8 per hour
2oo3:
PFD=P2 + P3 ≈ 5.48E-4
PFH=v2+v3=1.25E-7 per hour

For comparison:

1oo3:

$$PFD \approx \frac{(\lambda_{DU}\tau)^3}{4} + \frac{C_{1oo3}\beta\lambda_{DU}\tau}{2} \approx 1.1E-4$$

$$PFH \approx \lambda_{DU}^3\tau^2 + C_{1oo3}\beta\lambda_{DU} \approx \text{ 2.51E-8 per hour}$$

2oo3:

$$PFD \approx (\lambda_{DU}\tau)^2 + \frac{C_{2oo3}\beta\lambda_{DU}\tau}{2} \approx 5.15E-4$$

$$PFH \approx 3\lambda_{DU}^2\tau + C_{1oo3}\beta\lambda_{DU} \approx 1.26E-7 \text{ per hour}$$

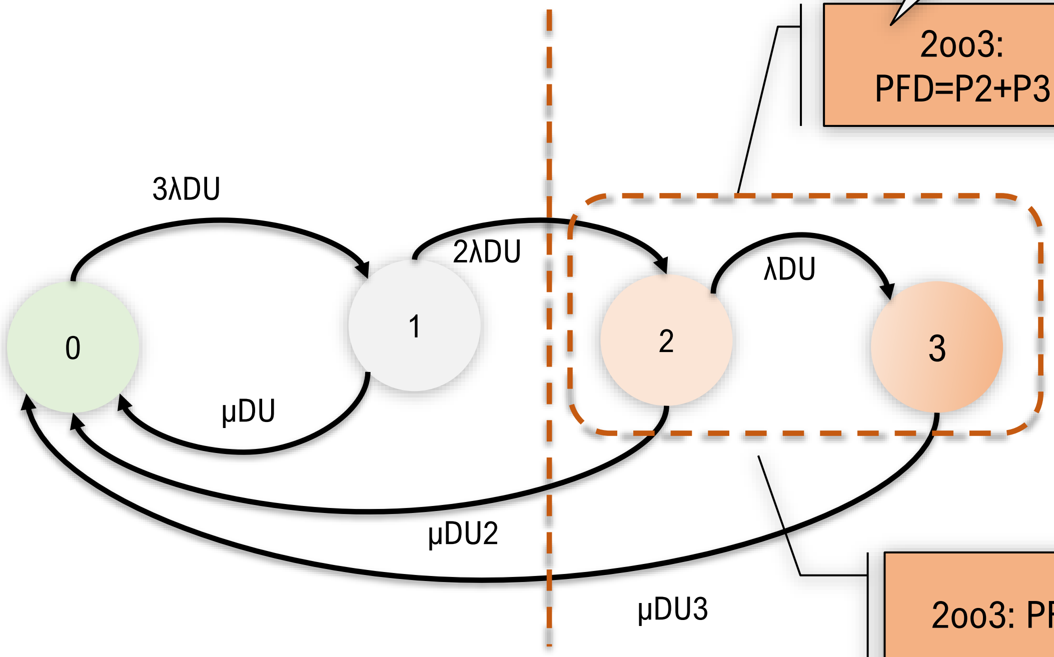
Markov gir en ganske lik verdi for 1oo3 votering og litt høyere for 2oo3 votering. Muligens noen flere transisjoner regnet inn til P2 tilstanden.

Miniprosjekt oppgave 4: a)-d) DU only

State	Description
0	3 PTer OK
1	1 PT DU, 2 PTer OK
2	2 PTer DU, 1PT OK
3	3 PTer DU

Make sure that you also understand how to expand equation for P2 and P3

2oo3: PFD=P2+P3



2oo3: PFH=v2+v3= P1*2λ_{DU}

a) Lag to Markovmodeller (steady state) for PT-ene: En modell for kun uavhengige DU feil og en modell med både DU feil og fellesfeil (standard betafaktormodellen) er tatt med og implementer den i excel arket som er oppgitt (det er også lov å kode via Python om noen ønsker det).

b) Forklar hvordan du kan regne ut PFD-bidraget for disse ved å benytte denne modellen.

c) Beregn PFD og kommenter på forskjellen i svaret du fikk for det samme delsystemet i oppgave b).

d) Vis hvordan du kan beregne PFH utfra samme deloppgave og utfør beregningen. Sammenlign med resultatet du fikk i oppgave b).

e) Anta at voteringen endres fra 2oo3 til 1oo3. Hvordan kan vil dette endre måten du regner ut PFD og PFH på? (vist i Markov modellen, trenger ikke å regne ut)

Parameter	Description	
λDU,PT	DU failure rate	0.5E-6
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval	2/8760
μDU2	3/τ which is the mean time to restore TWO DU failures, where τ is the test interval	3/8760
μDU3	4/τ which is the mean time to restore 3 failures	4/8760

Miniprosjekt oppgave 4: a)-d) DU only

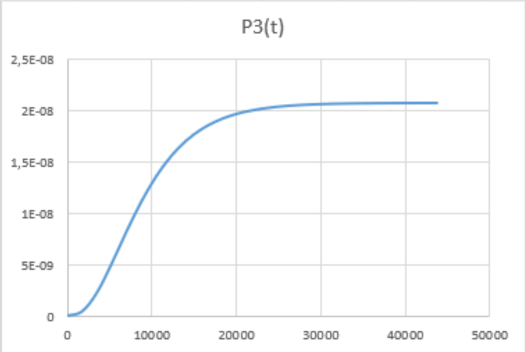
	Parameters	Value	Comment
1	Parameters	Value	Comment
2	nDim	4	Dim. of problem=r+1
3	MaxTime	43800	For time dep. solut.
4	Initial state	0	Initial state
5	State2Follow	3	State to plot
6	LDD	0,00E+00	Failure rate per hour
7	LDU	5,00E-07	Failure rate per hour
8	muDD	0	repair rate per hour
9	muDU	0,0002283	repair rate per hour
10	muDU2	0,0003425	repair rate per hour
11			
12	muDU3	0,0004566	repair rate [Steady state]
13	tau	8760	hours
14			
15			[Time dep.]
16			[Avg. prob.]
17			
18	**Open VBA Module:**		
19	Windows PC		Press <Alt> + F11
20	Mac		Press <fn> + <alt> + F11
21			
22			
23	**Modules:**		
24	mLib	Library of matrix functions	
25	MarkovLib	Utility functions, Markov	
26	WSlib	Utility functions, Worksheet R/W	
27	ButFunctions	Functions "behind" the buttons	
28			
29			
30	2oo3		
31	PFD=P2+P3		1,90E-05
32	PFH=P1*2*LDU=		6,50E-09 per hour
33	Simplified formulas:		
34	PFD (approx, DU only):		1,92E-05
35	PFH (approx, DU only)		6,57E-09
36	1oo3		
37	PFD=P3		2,07E-08
38	PFH=P2*LDU		9,47E-12 per hour
39	Simplified formulas:		
40	PFD (approx, DU only):		2,10E-08
41	PFH (approx, DU only)		9,59E-12

v_i	$P_i(t)$	P_i
1,49E-06	9,94E-01	9,93E-01
1,49E-06	5,86E-03	6,50E-03
6,5E-09	1,582E-05	1,8949E-05
9,45E-12	1,628E-08	2,0694E-08

A-Matrix==>

From ↓ / To -->	0	1	2	3	4	5	6
0		0,0000015	0	0			
1	0,000228		0,000001	0			
2	0,000342	0		5E-07			
3	0,000457	0	0				
4							
5							
6							
7							
8							
9							

t	$P_3(t)$
0	0
1752	4,2E-10
3504	2,2E-09
5256	5,1E-09
7008	8,2E-09
8760	1,1E-08
10512	1,4E-08
12264	1,6E-08
14016	1,7E-08
15768	1,8E-08
17520	1,9E-08
19272	1,9E-08
21024	2E-08
22776	2E-08
24528	2E-08
26280	2E-08
28032	2,1E-08
29784	2,1E-08
31536	2,1E-08
33288	2,1E-08
35040	2,1E-08
36792	2,1E-08
38544	2,1E-08
40296	2,1E-08
42048	2,1E-08
43800	2,1E-08
Average	**1,6E-08**



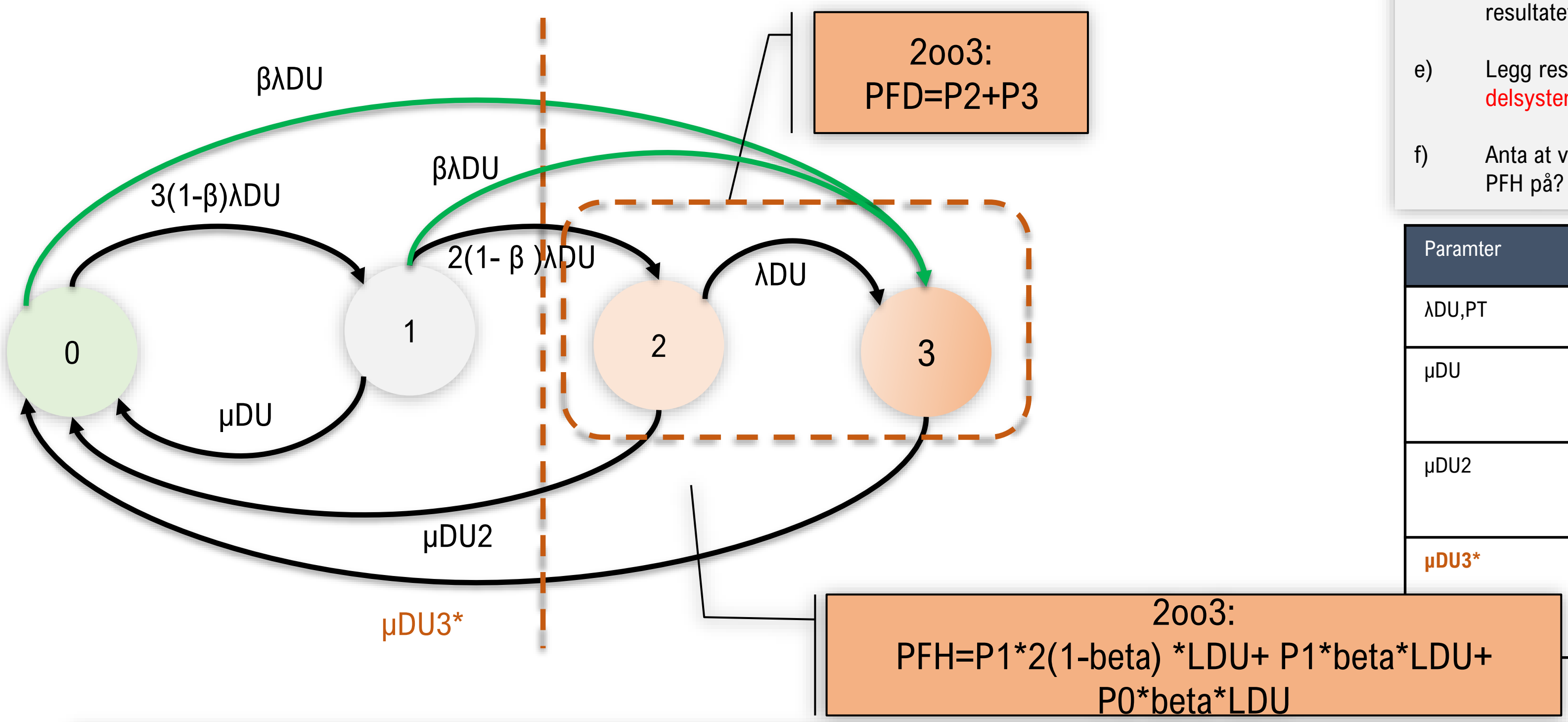
2oo3:

PFD=P2 + P3≈ 1,90E-5

PFH=2*LDU*P1 ≈6.50E-9 per hour

Miniprosjekt oppgave 4 : a)-d) DU and CCFs

State	Description
0	3 PTer OK
1	1 PT DU, 2 PTer OK
2	2 PTer DU, 1PT OK
3	3 PTer DU



a) Lag to Markovmodeller (steady state) for PT-ene: **En modell** for kun uavhengige DU feil **og en modell** med både DU feil og fellesfeil (standard betafaktormodellen) er tatt med og implementer den i excel arket som er oppgitt (det er også lov å kode via Python om noen ønsker det).

b) Forklar hvordan du kan regne ut PFD-bidraget basert på hver av de **to modellene**. Her kan dere sette opp ligningen med vektoren P (sannsynlighet i steady state for alle tilstander) og transisjonsmatrisen, men ikke løse ut ligningene.

c) Beregn PFD **for begge modellene** ~~og kommenter på forskjellen i svaret du fikk for det samme delsystemet i oppgave b)~~. Bruk excel-regnearket til utregningen (på muntlig eksamen kan du få spørsmål til hva de ulike delene i regnearket, og hva du legger inn og hva som beregnes).

d) Vis hvordan du kan beregne PFH utfra samme deloppgave og utfør beregningen. Sammenlign med resultatet du fikk i oppgave b). Bruk også her excel regnearket til utregningen.

e) Legg resultatene inn i skjema fra oppgave 2e) og kommenter i forhold til resultater for samme delsystem for seminar/oppgave 2 og 3.

f) Anta at voteringen endres fra 2oo3 til 1oo3. Hvordan kan vil dette endre måten du regner ut PFD og PFH på? Forklar med utgangspunkt i Markov modellen, uten å utføre noen utregninger.

Paramter	Description	
λDU,PT	DU failure rate	0.5E-6
μDU	2/τ which is the mean time to restore one DU failure, where τ is the test interval	2/8760
μDU2	3/τ which is the mean time to restore TWO DU failures, where τ is the test interval	3/8760
μDU3*	2/τ which is the mean time to restore CCF	2/8760

Miniprosjekt oppgave 4 : a)-d) DU and CCFs

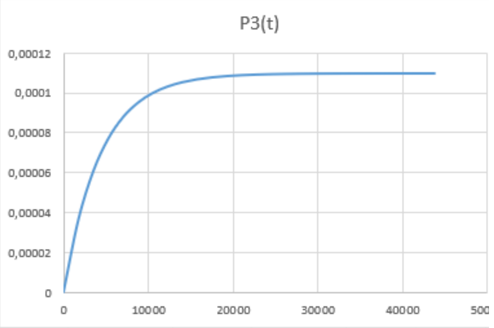
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Parameters	Value	Comment				A-Matrix==>			To -->					
2	nDim	4	Dim. of problem=r+1		v_i	$P_i(t)$	P_i			0	1	2	3	4	5
3	MaxTime	43800	For time dep. solut.		1,44E-06	9,94E-01	9,94E-01	From	0		0,000001425	0	0,000000025		
4	Initial state	0	Initial state		1,42E-06	5,57E-03	6,18E-03	↓	1	0,000228		9,5E-07	0,000000025		
5	State2Follow	3	State to plot		5,87E-09	1,43E-05	1,71E-05		2	0,000342	0		0,0000005		
6	LDD	0,00E+00	Failure rate per hour		2,5E-08	9,86E-05	1,10E-04		3	0,000228	0	0			
7	LDU	5,00E-07	Failure rate per hour						4						
8	muDD	0	repair rate per hour						5						
9	muDU	0,0002283	repair rate per hour						6						
10	muDU2	0,0003425	repair rate per hour						7						
11	muDU3	0,0002283	repair rate per hour						8						
12	beta	5 %	CCF fraction						9						
13	tau	8760	hours												

Steady state

Time dep.

Avg. prob.

	A	B	C	D	E	F	G	H
14							t	$P_3(t)$
15							0	0
16							1752	3,62E-05
17							3504	6,04E-05
18	Open VBA Module:						5256	7,66E-05
19	Windows PC		Press <Alt> + F11				7008	8,75E-05
20	Mac		Press <fn> + <alt> + F11				8760	9,48E-05
21							10512	9,97E-05
22							12264	1,03E-04
23	Modules:						14016	1,05E-04
24	mLib	Library of matrix functions					15768	1,07E-04
25	MarkovLib	Utility functions, Markov					17520	1,08E-04
26	WSlib	Utility functions, Worksheet R/W					19272	1,08E-04
27	ButFunctions	Functions "behind" the buttons					21024	1,09E-04
28							22776	1,09E-04
29							24528	1,09E-04
30	2oo3						26280	1,09E-04
31	PFD=P2+P3			1,27E-04			28032	1,09E-04
32	PFH=P0*beta*LDU+P1*beta*LDU+P1*2*(1-beta)*LD			3,09E-08	per hour		29784	1,09E-04
33	Simplified formulas:						31536	1,09E-04
34	PFD (approx, DU only):			1,27E-04			33288	1,09E-04
35	PFH (approx, DU only)			3,09E-08	per hour		35040	1,09E-04
36	1oo3						36792	1,09E-04
37	PFD=P3			1,10E-04			38544	1,10E-04
38	PFH=P0*beta*LDU+P1*beta*LDU+P2*LDU			2,5E-08	per hour		40296	1,10E-04
39	Simplified formulas:						42048	1,10E-04
40	PFD (approx, DU only):			1,10E-04			43800	1,10E-04
41	PFH (approx, DU only)			2,5E-08	per hour		Average	9,68E-05



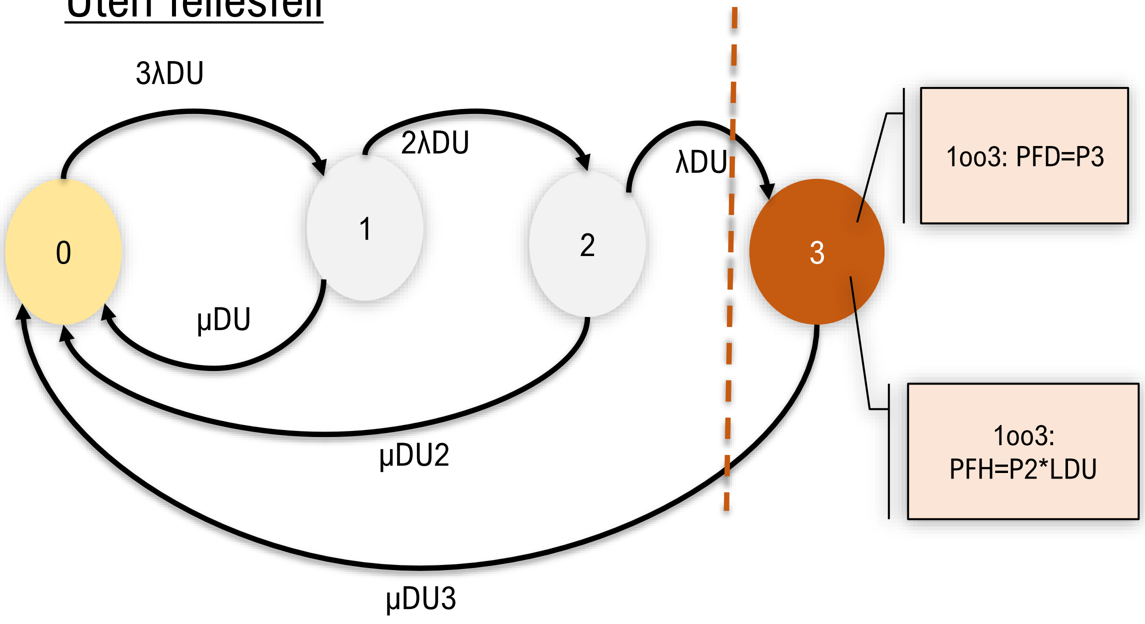
2oo3:

PFD=P2 + P3≈ 1.27E-4

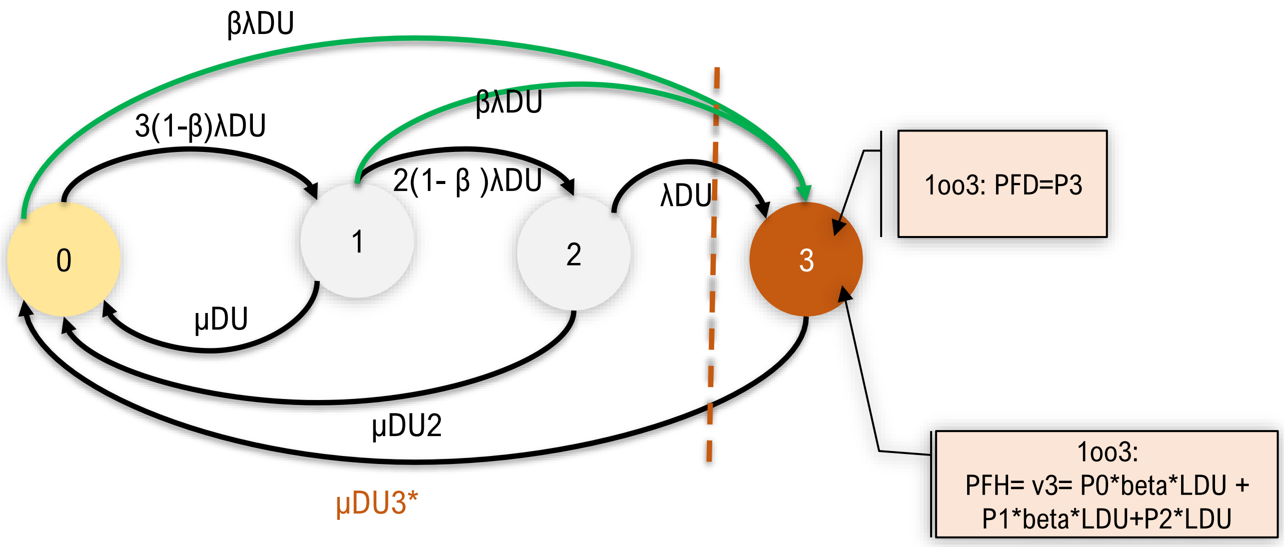
PFH ≈ 3.09E-8 per hour

Miniprosjekt oppgave 4 : e) If 1oo3 instead of 2oo3

Uten fellesfeil



Med fellesfeil



Miniprosjekt oppgave 2: e)

KUN for Subsystem input elements (PTs)

Result	Forenklede formler (oppgave 2)	Feiltreanalyse (oppgave 3)	Markov analyse (oppgave 4)
PFD			
PFD no CCFs	1.92E-5	1.92E-5 (1.43E-5*)	1,90E-5
PFD w CCFs	1.27E-4	1.27E-4 (1.22E-4*)	1.27E-4
PFD w CCF using PDS method	2.38E-4		
PFH (per time)			
PFH no CCFs	6.57E-9	6.57E-9	6.50E-9
PFH w CFFs	3.09E-8	3.09E-8	3.09E-8

*CARA FT